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ON CERTAIN IMPLICATIONS OF POSSIBLE CHANGES IN THE FORM AND DIMENSIONS OF THE SUN, AND SOME SUGGESTIONS TOWARD EXPLAINING CERTAIN PHENOMENA OF VARIABLE STARS[1]

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The shape of the sun and the possibility of changes in its dimensions have been matters of keen interest to astronomers for about a century. Among those who have contributed to the discussion of these questions we may mention Von Lindenau (1809), Bianchi (1831), Secchi (1871), Auwers (1873 and 1895), Newcomb and Holden (1874), Ambronn (1905), and Poor (1908). In the last of Poor's papers[2] there are full references and complete abstracts of all the most important discussions of the subject. It is sufficient to state here that up to Auwers' work in 1885 the observations were obtained with meridian instruments, that the later observations of Auwers and those of Ambronn were made with the heliometer, and that Poor's work, aside from a rediscussion of the heliometer measures of Auwers and Ambronn, was based upon twenty-two Rutherford plates taken in the years 1870–72, nine Northfield plates taken in 1893–94, and six Yerkes Observatory plates taken in 1907. In using these photographic plates, Poor limited his discussion to the question of the differences in the equatorial and polar diameters.

[1] This paper was prepared by the author as a research Associate of the Carnegie Institution. Since it was written Professor E. B. Frost called the writer's attention to the fact that Ritter had treated certain phases of the problems considered here, in his celebrated memoirs in *Wiedemanns Annalen*, Vols. **5-20** (1878–83). The methods employed by Ritter are altogether different from those used here. It will be noted below where the results are common.

[2] "An Investigation of the Figure of the Sun and of Possible Variations in Its Size and Shape," *Annals of the N. Y. Acad. Sci.*, **18**, No. 9, Part III, 385–424, 1908.

The results obtained in the various investigations have been most discordant. Von Lindenau reached the conclusion that there are periodic variations in the sun's diameter; from the same observation Auwers came to the opposite conclusion; Secchi found the diameter varying inversely with the number of sun-spots; Auwers, from the same data, found no variations; Newcomb and Holden reached the result that there is no long-term variation; in his first discussion of the heliometer observations Auwers found that the diameter varies directly as the number of sun-spots, and in a second discussion that the apparent variation was due to varying personal equations; and Ambronn was convinced that during the interval of thirteen years covered by his observations there was no periodic or secular change in the diameter of the sun exceeding 0".1.

The conclusions respecting the shape of the sun were equally varied, the polar diameter being found a little greater than the equatorial about as often as the opposite. Poor infers from all the evidence that "the exact shape of the sun is not known with certainty;" and that the difference in the polar and equatorial radii is probably not more than 0".25. He states that the heliometer observations show a fluctuation in the size of the sun of probably not over 0".10. Ambronn thinks there is no observational evidence of changes in its dimensions.[1]

In scientific work there is rightly no higher evidence than the observational. The facts derived from observations lead to theories and sometimes overthrow them. Yet theories and theoretical work may often guide observations, as in the case of the discovery of *Neptune*, or reveal inconsistencies and errors in them. At the present time theories are most apt to be of service in subjects of a dynamical character, for the foundations of mechanics are laid deep in an enormous mass of experience, and the fundamental principles of it have been shown to be consistent with the phenomena of the physical universe with a thoroughness and degree of accuracy approached in no other part of science. Now the questions of the form of the sun and the possible variations in its dimensions are of a dynamical nature, and may well be considered from the dynamical standpoint.

In considering the shape of the sun we may inquire:

I. What the character of the figure of equilibrium of such a mass is.

[1] *Astrophysical Journal*, **23**, 344, 1906.

II. The extent of its deviation from sphericity.

III. The character and periods of the oscillations it may undergo if its equilibrium is disturbed.

In considering the possibility of changes in the dimensions and shape of the sun we may inquire:

IV. The effect upon its rate of rotation;

V. The effect upon its energy of rotation;

VI. The effect upon its potential energy;

VII. The effects upon its temperature and rate of radiation.

VIII. Whether the results may not be used to explain some of the phenomena of variable stars of certain types.

We shall consider these questions briefly in this order.

I. THE FIGURE OF THE SUN

If the sun were a homogeneous liquid its figure of equilibrium for steady rotation (omitting certain theoretical figures having, for its density and rate of rotation, quite different shapes) would be strictly an oblate spheroid whose degree of oblateness would depend upon its density and rate of rotation. It is practically certain that the density of the sun increases below its surface, at least as far as it remains approximately gaseous in character, and the increase in density may continue even to its center. If this central condensation were slight the form of equilibrium would remain nearly an oblate spheroid. It is known from a consideration of the limiting case of an atmosphere of vanishing mass surrounding a rigid spherical center, and from the investigations of Clairaut,[1] that a heterogeneous body of this character will be less oblate than a homogeneous mass of the same mean density and having the same rate of rotation. This is verified by actual observations in the case of the earth, *Jupiter*, and *Saturn*, the difference being very marked in the case of *Saturn*.

On the other hand, a prolate spheroid is not a figure of equilibrium for the homogeneous fluid. Neither is it a figure of equilibrium for a fluid mass whose density increases continuously toward its center of gravity. Therefore, if the sun is a figure of equilibrium we are justified in concluding that it is oblate and approximately spheroidal in form, and that its oblateness is less than that of the corresponding homogeneous mass.

[1] Tisserand, *Mécanique céleste*, **2**, chap. xiii.

II. THE OBLATENESS OF THE SUN

The relation between the rate of rotation, density, and oblateness of the homogeneous spheroid is the well-known equation[1]

$$\frac{\omega^2}{2\pi k^2\sigma}=\frac{1}{\lambda^2}\left\{(3+\lambda^2)\frac{\tan^{-1}\lambda}{\lambda}-3\right\}, \tag{1}$$

where ω is is the angular rate of rotation, k^2 the gravitational constant, and σ the density. Let a represent the equatorial radius, b the polar radius, and then λ is defined by $a^2=b^2(1+\lambda^2)$. Let $\frac{\omega^2}{2\pi k^2\sigma}=\eta$ and expand the right member. Then we get

$$\eta=\tfrac{4}{15}\lambda^2-\tfrac{8}{35}\lambda^4+\tfrac{4}{21}\lambda^6\ldots \tag{1'}$$

Inverting this series we have

$$\lambda^2=\tfrac{5}{4}\eta+\tfrac{6}{7}(\tfrac{5}{4})^2\eta^2+\tfrac{3}{4}\tfrac{7}{9}(\tfrac{5}{4})^3\eta^3\ldots, \tag{2}$$

a convenient formula for computing λ^2 when η is small.

Taking the mean solar day as the unit of time, the mass of the sun as the unit of mass, the mean distance from the earth to the sun as the unit of distance, the period of the sun's rotation as 25.3 days, and its density as 1.41 times that of water, we find in the case of the sun

$$\eta=2\times10^{-5}.$$

Hence equation (2) gives

$$\lambda^2=75\times10^{-6}.$$

If we let ϵ represent the ellipticity we have

$$\epsilon=\frac{a-b}{a}=1-\frac{1}{\sqrt{1+\lambda^2}}=\frac{\lambda^2}{2}-\tfrac{3}{8}\lambda^4\ldots=37\times10^{-6}.$$

The actual ellipticity of the sun must be less than this quantity. The difference in the equatorial and polar diameters of the sun must be less than

$$2a-2b=2b(\sqrt{1+\lambda^2}-1)=b(\lambda^2-\tfrac{1}{4}\lambda^4\ldots). \tag{3}$$

Taking $2b$, as seen from the earth, equal to 1920″ we find that the difference in equatorial and polar diameters of the sun must be less than 0″.07 as seen from the earth. Even if only the order of this result is correct, the quantity in question is beyond observation with present means.

[1] Tisserand, *op. cit.*, chap. vi.

III. OSCILLATIONS OF THE SUN

Consider an incompressible homogeneous liquid in a stable figure of equilibrium. For the moment we shall neglect the rotation and suppose the figure is a sphere. If it is deformed slightly and left to its own gravitation it will oscillate around the spherical shape, the amplitude of the oscillation depending upon the extent of the original deformation, and its period upon the character of the deformation. It is a well-known theorem that any slight deformation of a sphere may be represented by a converging series of spherical harmonics. The period of oscillation of each harmonic term will depend upon the order of the harmonic. The problem of determining the characteristics of this sort of gravitational oscillations of a sphere was first solved by Lord Kelvin,[1] and he showed that the period of oscillation of the harmonic of order n is

$$T_n = 2\pi \sqrt{\frac{2n+1}{2n(n-1)}} \sqrt{\frac{a}{g}}, \tag{4}$$

where a is the radius and g the surface-gravity. Since g is directly proportional to the density and first power of the radius it follows that T_n depends only upon the order of the harmonic and the inverse one-half power of the density.

The first value of n having physical application to the problem under consideration is $n=2$, and the period of the corresponding term is

$$T_2 = 2\pi \sqrt{\tfrac{1}{4}} \sqrt{\frac{a}{g}} = 3^{\mathrm{h}}\ 8^{\mathrm{m}} \tag{5}$$

for a homogeneous mass having the dimensions and mean density of the sun. The higher the order of the harmonic the shorter the period of oscillation.

In considering the oscillations of a viscous spheroid Professor Lamb has shown that the time it takes the amplitude of a harmonic oscillation to decrease to $\frac{1}{e} = 0.37\ \ldots$ of its original value is[2]

$$\tau = \frac{1}{(n-1)(2n+1)} \cdot \frac{a^2}{\nu}, \tag{6}$$

where $\nu = \frac{\mu}{\sigma}$, σ being the density and μ the coefficient of viscosity.

[1] *Phil. Trans.*, **153**, 612, 1863.

[2] *Proceedings of the London Math. Soc.*, **13**, 61, 1881–82.

For water at moderate temperatures $\nu = 0.014$; then, taking $n = 2$, as in applying (4), equation (6) gives for a body having the size and mass of the sun $\tau = 22 \times 10^{14}$ years. The rate at which friction destroys the oscillations increases as the order of the harmonic increases. In a body having the size and mass of the sun a gravitational oscillation represented by a harmonic of low order, once started, will persist for an extremely long time.

But these results are not applicable to the sun without modification. In the first place the sun is undoubtedly much condensed toward its center, and this will have an effect on the period of the gravitational oscillations. Because of the increased mean potential the period is decreased by this factor. In the second place the material is compressible and probably highly elastic as regards compressibility. This factor probably will also somewhat decrease the period of an oscillation.[1] These considerations lead to the conclusion that if the sun is undergoing oscillations, their periods are only a few hours, though separate oscillations of different periods may unite at long intervals like "beats" in musical notes. It is not improbable that the sun is yet undergoing slight oscillations started by some ancient disturbance.

IV. EFFECT OF OSCILLATIONS UPON THE RATE OF ROTATION

The hypothesis is explicitly made here that the sun is not now subject to sensible external disturbances. Hence all changes in its dimensions and rotation must come from internal forces, and it follows that its moment of momentum will be constant. The formula for the moment of momentum, M, is

$$M = CmA^2\omega ,$$

where m is the mass, A one of the linear dimensions, ω the angular rate of rotation, and C a constant depending upon the shape of m, the definition of A, and the distribution of density. For simplicity we shall suppose that the sun is always homogeneous, though it is almost certainly densest far below its surface. We shall suppose also that its axis of rotation is always an axis of symmetry. The general

[1] In Emden's *Gaskugeln*, which was received after this paper was in type, the computation is made (p. 452) under a number of simplifying assumptions, and the period of oscillation for the sun is found to be two hours.

character of the results cannot be changed by this assumption. Then, whatever may be its shape at any time, the distance, r, from its center of gravity to its surface can be expressed uniquely in the form

$$r=a[1+p_0+p_1P_1(\phi)+p_2P_2(\phi)+\dots], \tag{7}$$

where ϕ is the colatitude, $P_n(\phi)$ Legendre's coefficient of the nth order, and p_n numbers depending upon its shape. If the mass changes in shape, the $P_n(\phi)$ remain fixed while the p_n vary. In particular, if the mass undergoes periodic oscillations the p_n vary periodically, and, so far as first-order disturbances are concerned, independently and with different periods. A change in p_0 implies a simple spherical dilatation,[1] while a change in the other p_n involves a change in shape as well as in volume. For example, the sphere modified by the term depending on p_2 is prolate or oblate according as p_2 is positive or negative, while the term depending on p_3 deforms the sphere into an egg-shaped figure. The periods of the oscillations depend upon many uncertain factors such as the elasticity of the sun, the distribution of its density and temperature, and the effects of the enormous internal stresses to which its matter is subject. On this account it is difficult, if not impossible, to determine the periods of the p_n in the actual case with any degree of certainty. But the exact periods are not important in the present connection. The work of Kelvin gives their general order of magnitude.[2]

Since the origin has been taken at the center of gravity we have

$$\int \rho \cos\phi\, dm=\sigma\int_0^{2\pi}\int_0^{\pi}\int_0^{r}\rho^3\cos\phi\sin\phi\, d\rho\, d\phi\, d\theta=0, \tag{8}$$

where ρ is the distance from the center of gravity to the element of mass dm. Integrating with respect to ρ, substituting for r from

[1] Ritter treated infinitesimal dilatational oscillations under the hypotheses that the mass is always spherical and homogeneous, and that it obeys the laws of gases in its expansion and contraction, *op. cit.*, **8**, § 25, 174 (1879); and finite oscillations under the same hypotheses, *op. cit.*, **13**, § 41 (1881). His analysis did not cover the more important oscillations specified by p_1P_1, p_2P_2, ...

[2] Under his hypotheses, Ritter found for the sun $T=5$h. 50m. *op. cit.*, **8**, 178, (1879). The dilatational oscillations were not included in Kelvin's analysis, for he treated only incompressible fluids.

equation (7), and making use of the properties of the integrals of spherical harmonics, we have up to terms of the order $p_2 p_3$,

$$p_1 = -\tfrac{3}{5}\tfrac{7}{5} p_2 p_3 \cdots . \tag{9}$$

Therefore p_1 is small relatively to p_2 and p_3 and it may be neglected. As was seen from equation (6) the terms depending upon harmonics of higher orders are most quickly reduced by friction. Therefore it will be unnecessary to go to terms of high order.

Since the continuity condition of incompressible fluids has not been imposed, the volume and mean density will in general change with the shape as well as with the dilatation expressed through p_0. Letting v represent the volume, we have

$$v = \frac{1}{\sigma}\int dm = \int_0^{2\pi}\int_0^{\pi}\int_0^{r} \rho^2 \sin\phi \, d\rho d\phi d\theta ,$$

which becomes after integrating, as above,

$$v = \tfrac{4}{3}\pi a^3 (1+p_0)^3 \left[1 + \frac{3p_2^2}{5(1+p_0)^2} + \frac{3p_3^2}{7(1+p_0)^2} + \frac{2p_2^3}{35(1+p_0)^3} + \cdots\right]. \tag{10}$$

We shall now compute the moment of inertia of this mass with respect to its axis of symmetry, which we shall assume is also its axis of rotation. It is defined by the equation

$$I = \frac{M}{\omega} = \sigma \int_0^{2\pi}\int_0^{\pi}\int_0^{r} \rho^4 \sin^3\phi \, d\rho d\phi d\theta .$$

Making use of the fact that $\sin^2\phi = \tfrac{2}{3}(1 - P_2)$ and carrying out the integration as in the preceding cases, we find

$$\frac{M}{\omega} = \tfrac{8}{15}\pi\sigma a^5 (1+p_0)^5 \left[1 - \frac{p_2}{1+p_0} + \frac{2p_2^2}{(1+p_0)^2} + \tfrac{10}{7}\frac{p_3^2}{(1+p_0)^2} + \cdots\right]. \tag{11}$$

Making use of (10) this equation becomes

$$M = \tfrac{2}{5} m a^2 (1+p_0)^2 \left[1 - \frac{p_2}{1+p_0} + \tfrac{7}{5}\frac{p_2^2}{(1+p_0)^2} + \frac{p_3^2}{(1+p_0)^2} + \cdots\right]\omega ,$$

or, with sufficient accuracy when p_0, p_2, . . . are small,

$$M = \tfrac{2}{5} m a^2 [1 + 2p_0 + p_0^2 - p_2 - p_0 p_2 + \tfrac{7}{5}p_2^2 + p_3^2 + \cdots]\omega . \tag{12}$$

In astronomical units for the sun, $m = 1$, $a = 466 \times 10^{-5}$, $\omega = \dfrac{2\pi}{25 \cdot 3}$, and p_0, p_2, . . . are very small. Hence equation (12) gives $M = 216 \times 10^{-8}$ for the sun.

Oscillations in the sun either in dimensions or in form will change

its rate of rotation. We may inquire whether the possible direct changes in dimensions and form, or the consequences of these changes upon the rate of rotation, may the more easily be observed. We note immediately that the gravitational and elastic oscillations have short periods, and that there are no permanent markings on the sun from which its period can be accurately determined. Consequently we see that this indirect process is probably doomed to failure. But we may still inquire whether the possible secular shrinking may not more quickly be detected by observations of the acceleration of its rotation than by direct measurements of its dimensions. If we represent its period by P, we have from (12), when $p_0=p_2=\ldots=0$

$$P=\frac{2\pi}{\omega}=\frac{4\pi ma^2}{5M}.$$

Suppose the radius shrinks the fraction of its diameter p_0. Then the period of rotation becomes

$$P'=\frac{4\pi ma^2(1-p_0)^2}{5M}.$$

Suppose the shrinkage in diameter is 1".0. Then

$$p_0=\tfrac{1}{1920}$$

and we have

$$P'-P=-\frac{8\pi ma^2}{5M}p_0\left(1-\frac{p_0}{2}\right)=-0.0265\text{d.}=-38\text{ minutes.} \qquad (13)$$

That is, the diminution of period is so small that it could not be determined with certainty, and we conclude that the change in diameter will be observed before the change in the rate of rotation.

V. EFFECTS OF OSCILLATIONS OF THE SUN UPON ITS ENERGY OF ROTATION[1]

The rotational energy of the sun is

$$E=\tfrac{1}{2}I\omega^2=\tfrac{1}{2}M\omega. \qquad (14)$$

As a consequence of (12) this reduces to

$$E=\frac{5M^2}{2ma^2}[1-2p_0+3p_0^2+p_2-3p_0p_2-\tfrac{7}{5}p_2^2-p_3^2\ldots]. \qquad (15)$$

[1] Ritter recognized this factor, 13, 370, but made no discussion of its importance, even in the case of simple expansion and contraction which he considered.

The sun is rotating so slowly that the changes in its energy of rotation produced by any admissible oscillations will be found in the applications which follow to be relatively unimportant.

VI. EFFECTS OF OSCILLATIONS OF THE SUN UPON ITS SELF-POTENTIAL

We shall now compute the potential of the whole mass upon itself. Representing this potential by W we have

$$W=\tfrac{1}{2}k^2\int dm\int\frac{dm'}{\Delta}, \tag{16}$$

where dm and dm' are any two distinct elements of mass, Δ the distance between them, and where the integrals must separately be extended over the entire body. We shall first compute

$$V=\int\frac{dm'}{\Delta}. \tag{17}$$

Let the co-ordinates of dm' be ρ', ϕ', θ' and those of dm be ρ, ϕ, θ. Then

$$\left.\begin{aligned} dm'&=\sigma\rho'^2\sin\phi' d\rho' d\phi' d\theta',\\ \Delta^2&=\rho^2+\rho'^2-2\rho\rho'\cos\gamma, \text{ where}\\ \cos\gamma&=\cos\phi\cos\phi'+\sin\phi\sin\phi'\cos(\theta-\theta'). \end{aligned}\right\} \tag{18}$$

Then we have

$$\left.\begin{aligned} V&=V_1+V_2, \text{ where}\\ V_1&=\sigma\int_0^{2\pi}\int_0^{\pi}\int_0^{\rho}\frac{\rho'^2\sin\phi'}{\Delta}d\rho' d\phi' d\theta',\\ V_2&=\sigma\int_0^{2\pi}\int_0^{\pi}\int_{\rho}^{\infty}\frac{\rho'^2\sin\phi'}{\Delta}d\rho' d\phi' d\theta'. \end{aligned}\right\} \tag{19}$$

In the region of integration covered in V_1 we have

$$\Delta^{-1}=\frac{1}{\rho}\left[1+\frac{\rho'}{\rho}Y_1+\left(\frac{\rho'}{\rho}\right)^2Y_2+\ldots\right], \tag{20}$$

and in the region of integration covered in V_2,

$$\Delta^{-1}=\frac{1}{\rho'}\left[1+\frac{\rho}{\rho'}Y_1+\left(\frac{\rho}{\rho'}\right)^2Y_2+\ldots\right], \tag{21}$$

where $Y_n(\phi, \phi', \theta, \theta')$ is Laplace's coefficient of the nth order. Consequently

$$V_1=\sigma\int_0^{2\pi}\int_0^{\pi}\rho^2\left[\tfrac{1}{3}+\frac{Y_1}{4}+\ldots+\frac{Y_n}{n+3}+\ldots\right]\sin\phi' d\phi' d\theta',$$

which, by the properties of the integrals of spherical harmonics, becomes

$$V_1=\tfrac{4}{3}\pi\sigma\rho^2. \tag{22}$$

Similarly V_2 becomes

$$V_2=\sigma\int_0^{2\pi}\int_0^{\pi}\left[\frac{\rho'^2}{2}+\rho\rho' Y_1+\rho^2\log\rho' Y_2+\ldots + \frac{\rho^n Y_n}{(-n+2)\rho'^{n-2}}+\ldots\right]_\rho \sin\phi' d\phi' d\theta'.$$

Substituting the limits, this equation may be written

$V_2=V_2^{(1)}+V_2^{(2)}$, where

$$\left.\begin{aligned} V_2^{(1)}&=-\sigma\int_0^{2\pi}\int_0^{\pi}\left[\frac{\rho^2}{2}+\rho^2 Y_1+\rho^2\log\rho Y_2+\ldots + \frac{\rho^2}{-n+2}Y_n+\ldots\right]\sin\phi' d\phi' d\theta',\\ V_2^{(2)}&=\sigma\int_0^{2\pi}\int_0^{\pi}\left[\frac{r^2}{2}+\rho r Y_1+\rho^2\log r Y_2+\ldots + \frac{\rho^n r^{-n+2}}{-n+2}Y_n+\ldots\right]\sin\phi' d\phi' d\theta'. \end{aligned}\right\} \tag{23}$$

We find at once

$$V_2^{(1)}=-2\pi\sigma\rho^2. \tag{24}$$

The value of r from (15) must be substituted in the expression for $V_2^{(2)}$. Making use of the general equations

$$\int_0^{2\pi}\int_0^{\pi}P_{n_1}(\phi')Y_{n_2}(\phi,\phi',\theta,\theta')\sin\phi' d\phi' d\theta'=0,\ n_1\neq n_2,$$

$$\int_0^{2\pi}\int_0^{\pi}P_n(\phi')Y_n(\phi,\phi',\theta,\theta')\sin\phi' d\phi' d\theta'=\frac{4\pi}{2n+1}P_n(\phi),$$

$$\log r=\log a(1+p_0)+\sum_{i=1}^{\infty}(-1)^{i+1}\left[\frac{p_2}{1+p_0}P_2+\frac{p_3}{1+p_0}P_3+\ldots\right]^i,$$

we get

$$V_2^{(2)}=A+B+C+D,$$

where

$$A=\frac{\sigma}{2}a^2(1+p_0)^2\int_0^{2\pi}\int_0^{\pi}\left[1+\frac{p_2}{1+p_0}P_2+\frac{p_3}{1+p_0}P_3+\ldots\right]^2\sin\phi' d\phi' d\theta'$$

$$=2\pi\sigma a^2(1+p_0)^2\left[1+\frac{p_2^2}{5(1+p_0)^2}+\frac{p_3^2}{7(1+p_0)^2}+\ldots\right],$$

$$B=\sigma\rho a(1+p_0)\int_0^{2\pi}\int_0^{\pi}\left[1+\frac{p_2}{1+p_0}P_2+\ldots\right]Y_1\sin\phi' d\phi' d\theta'$$

$=0$ except for terms involving p_1,

$$C=\sigma\rho^2\int_0^{2\pi}\int_0^{\pi}\Big\{\log a(1+p_0)+\sum(-1)^{i+1}\Big[\frac{p_2}{1+p_0}P_2+\frac{p_3}{1+p_0}P_3+\ldots\Big]^i\Big\}Y_2\sin\phi'd\phi'd\theta'$$

$$=\frac{4\pi}{5}\sigma\rho^2\frac{p_2}{1+p_0}P_2+\ldots,$$

$$D=\frac{-\sigma\rho^n}{(n-2)a^{n-2}(1+p_0)^{n-2}}\int_0^{2\pi}\int_0^{\pi}\Big[1+\frac{p_2}{1+p_0}P_2+\frac{p_3}{1+p_0}P_3+\ldots+\frac{pn}{1+p_0}P_n+\ldots\Big]^{-n+2}Y_n\sin\phi'd\phi'd\theta',\ n\gtrless 3,$$

$$=\frac{4\pi\sigma\rho^n}{(2n+1)a^{n-1}(1+p_0)^{n-1}}p_nP_n(\phi)+\ldots,\ n\gtrless 3\ .\ .$$

Collecting results, we have finally

$$\left.\begin{aligned}V=&-\tfrac{2}{3}\pi\sigma\rho^2+2\pi\sigma a^2(1+p_0)^2\Big[1+\frac{p_2^2}{5(1+p_0)^2}+\frac{p_3^2}{7(1+p_0)^2}+\ldots\Big]\\&+\frac{4\pi}{5}\sigma\rho^2\frac{p_2}{1+p_0}P_2(\phi)+\frac{4\pi}{7}\frac{\sigma\rho^3}{a(1+p_0)}\frac{p_3}{1+p_0}P_3(\phi)+\ldots.\end{aligned}\right\}\quad(25)$$

Therefore (18) becomes

$$W=\tfrac{1}{2}k^2\int Vdm=\tfrac{1}{2}k^2\sigma\int_0^{2\pi}\int_0^{\pi}\int_0^{\rho}V\rho^2\sin\phi d\rho d\phi d\theta\ .$$

Substituting the value of V from (25), performing the integration with respect to ρ, making use of (7), and completing the integration, we find

$$W=\tfrac{16}{15}\pi^2k^2\sigma^2a^5(1+p_0)^5\Big[1+\tfrac{4}{5}\frac{p_2^2}{(1+p_0)^2}+\tfrac{2}{4}\tfrac{5}{9}\frac{p_3^2}{(1+p_0)^2}+\ldots\Big].\quad(26)$$

Since we are supposing that the density is uniform we have, making use of (10),

$$W=\tfrac{3}{5}k^2\frac{m^2}{a(1+p_0)}\Big[1-\tfrac{2}{5}\frac{p_2^2}{(1+p_0)^2}-\tfrac{1}{4}\tfrac{7}{9}\frac{p_3^2}{(1+p_0)^2}\cdots\Big],$$

or, with sufficient approximation,

$$W=\tfrac{3}{5}k^2\frac{m^2}{a}[1-p_0-\tfrac{2}{5}p_2^2-\tfrac{1}{4}\tfrac{7}{9}p_3^2\ldots].\quad(27)$$

The first term of this expression is, of course, the potential of a sphere upon itself with respect to infinite dispersion as the standard of reference.[1]

VII. EFFECTS OF OSCILLATIONS OF THE SUN UPON ITS TEMPERATURE AND RATE OF RADIATION

Let T represent the absolute temperature of the sun, s its specific heat, and H its rate of radiation. For simplicity we shall suppose T and s are uniform throughout the whole mass, and that s is constant with respect to the time. Let K represent the kinetic energy of the oscillations. Consider any two epochs t_1 and t_2 and let the various quantities at these epochs be distinguished by the corresponding subscripts. Then we have by the law of the conservation of energy

$$E_1 - W_1 + msT_1 + K_1 = E_2 - W_2 + msT_2 + K_2 + \int_{t_1}^{t_2} H dt \,. \tag{28}$$

Dividing by $t_2 - t_1$ and passing to the limit as $t_2 - t_1 \doteq 0$, and then making use of equations (15) and (27), we get

$$\frac{dT}{dt} = \tfrac{3}{5}\frac{k^2 m}{a}\left\{ -\frac{dp_0}{dt} - \tfrac{2}{5}\frac{dp_2^2}{dt} - \tfrac{1}{4}\tfrac{7}{9}\frac{dp_3^2}{dt} \cdots \right\} + \frac{5M^2}{2sm^2a^2}\left\{ 2\frac{dp_0}{dt} - 3\frac{dp_0^2}{dt} - \frac{dp_2}{dt} \cdots \right\} - \frac{1}{ms}\frac{dK}{dt} - \frac{1}{ms}H \,. \tag{29}$$

In order to treat this equation we must express H in terms of T. Stefan's law of radiation for a black body is that H is proportional to the product of the radiating surface and the fourth power of the absolute temperature. While it is not to be supposed that this law applies strictly to a star, still it seems probable that it will give the order of the true results. Hence we may write

$$H = cST^4 \,, \tag{30}$$

where c is a constant depending upon the units, and S is the whole surface of the radiating body.

If we represent an element of the surface by ds, we have

$$S = \int ds = \int_0^{2\pi} \int_0^{\pi} r \sqrt{r^2 + \left(\frac{dr}{d\phi}\right)^2} \sin\phi \, d\phi \, d\theta \,. \tag{31}$$

[1] Moulton, *Introduction to Celestial Mechanics*, pp. 37 ff. (49).

By means of equation (7), and the general formula

$$\frac{dP_n}{d\phi}=-[(2n-1)P_{n-1}+(2n-5)P_{n-3}+(2n-9)P_{n-5}+\dots]\sin\phi\,,$$

equation (31) becomes

$$S=\frac{a^2}{(1+p_0)^2}\int_0^{2\pi}\int_0^{\pi}\Big\{1+\frac{2p_2}{1+p_0}P_2+\frac{2p_3}{1+p_0}P_3+\frac{p_2^2}{2(1+p_0)^2}[2P_2^2+9P_1^2\sin^2\phi]$$
$$+\frac{p_2p_3}{(1+p_0)^2}[2P_2P_3+3P_1(1+5P_2)\sin^2\phi]+$$
$$\tfrac{1}{2}\frac{p_3^2}{(1+p_0)^2}[2P_3^2+(1+5P_2)^2\sin^2\phi]+\dots\Big\}\sin\phi\,d\phi\,d\theta\,. \qquad (32)$$

Making use of the fact that $\sin^2\phi=\frac{2}{3}(1-P_2)$ and of the properties of the integrals of spherical harmonics, this equation reduces to

$$S=4\pi a^2\{1-2p_0+\tfrac{4}{5}p_2^2+p_3^2+\dots\}\,. \qquad (33)$$

Substituting (30) and (33) in (29), we have

$$\frac{dT}{dt}=\tfrac{3}{5}\frac{k^2m}{sa}\Big\{-\frac{dp_0}{dt}-\tfrac{2}{5}\frac{dp_2^2}{dt}-1\tfrac{7}{9}\frac{dp_3^2}{dt}\cdots\Big\}$$
$$+\frac{5M^2}{2sm^2a^2}\Big\{2\frac{dp_0}{dt}-3\frac{dp_0^2}{dt}-\frac{dp_2}{dt}\cdots\Big\}$$
$$-\frac{1}{ms}\frac{dK}{dt}-\frac{4\pi a^2c}{ms}\Big\{1-2p_0+\tfrac{4}{5}p_2^2+p_3^2\dots\Big\}T^4\,. \qquad (34)$$

In a body like the sun c is very small. Hence for intervals of not more than a few weeks T may be integrated as a converging series of the form

$$T=u_0+u_1c+u_2c^2+\dots\,. \qquad (35)$$

Substituting (35) in (34), and integrating, we find

$$u_0=T_0+\tfrac{3}{5}\frac{k^2m}{sa}\Big\{-[p_0(t)-p_0(t_0)]-\tfrac{2}{5}[p_2^2(t)-p_2^2(t_0)]-1\tfrac{7}{9}[p_3^2(t)-p_3^2(t_0)]+\dots\Big\}$$
$$+\frac{5M^2}{2sm^2a^2}\Big\{2[p_0(t)-p_0(t_0)]-[p_2(t)-p_2(t_0)]-\frac{1}{ms}[K(t)-K(t_0)]\Big\}\,, \qquad (36)$$
$$u_1=-\frac{4\pi a^2}{sm}\int_{t_0}^{t}\Big\{1-2p_0(t)+\tfrac{4}{5}p_2^2(t)+p_3^2(t)+\dots\Big\}u_0^4dt\,,$$
$$u_2=-\frac{16\pi a^2}{sm}\int_{t_0}^{t}\Big\{1-2p_0(t)+\tfrac{4}{5}p_2^2(t)+p_3^2(t)+\dots\Big\}u_0^3u_1dt$$

where T_0 is the temperature at $t=t_0$. Equation (30) becomes

$$H=cS[u_0+u_1c+\dots]^4\,. \qquad (37)$$

We shall apply these equations to the sun for those short-period oscillations which alone can exist. There may be, of course, a secular contraction or expansion which might be accounted for through p_0. Since the present radiation of the sun could lower its temperature at the most only a very few degrees in a year, we may for intervals of not more than a few days regard T as equal to u_0. As was seen following equation (12) $\frac{M^2}{m^2}$ for the sun is very small relatively to the second term of u_0, and the term involving this factor may be neglected. Then the expression for T becomes with sufficient approximation for applications to the sun

$$T=T_0+\tfrac{3}{5}\frac{k^2m}{sa}\Big\{-[p_0(t)-p_0(t_0)]-\tfrac{2}{5}[p_2^2(t)-p_2^2(t_0)] \\ -\tfrac{1}{4}\tfrac{7}{9}[p_3^2(t)-p_3^2(t_0)]+\ldots\Big\}-\frac{1}{ms}[K(t)-K(t_0)]\,, \qquad (38)$$

and the rate of radiation is[1]

$$H=4\pi a^2c\Big\{T_0+\Big[-\tfrac{3}{5}\frac{k^2m}{sa}+\frac{T_0}{2}\Big][p_0(t)-p_0(t_0)]+ \\ \tfrac{1}{5}\Big[-\tfrac{6}{5}\frac{k^2m}{sa}+T_0\Big][p_2^2(t)-p_2^2(t_0)]+\Big[\tfrac{-51}{245}\frac{k^2m}{sa}+\frac{T_0}{4}\Big][p_3^2(t)-p_3^2(t_0)]+ \\ \ldots-\frac{1}{ms}[K(t)-K(t_0)]\Big\}^4. \qquad (39)$$

We shall now reduce these equations to numbers. For this purpose we shall select as units of mass, distance, and time, the kilogram, meter, and second, respectively. We shall suppose that s has the high value unity. (A smaller value would be more favorable to our most important conclusion.) Letting r_E and m_E represent the radius and mass of the earth, and g the value of gravity at its surface, we have

$$g=9.80\,, \qquad \frac{m}{m_E}=324{,}439\,,$$
$$r_E=6{,}371{,}000\,, \qquad a=692{,}428{,}000\,,$$
$$k^2=g\frac{r_E^2}{m_E}\,, \qquad \text{Calory}=425g\,.$$

[1] Ritter gave the formula for the dilatational term, *op. cit.*, **13**, 367.

Consequently equation (38) gives, since for $s=1$ the number of calories of heat per unit mass equals the temperature,

$$T=T_0+27\times10^6\left\{-[p_0(t)-p_0(t_0)]-\tfrac{3}{5}[p_2^2(t)-p_2^2(t_0)]-\right.$$

$$\left.\tfrac{1}{4}\tfrac{7}{5}[p_3^2(t)-p_3^2(t_0)]+\ldots\right\}-\frac{K(t)-K(t_0)}{ms}. \quad (40)$$

If the sun were denser at its center and if the law of density were preserved during the oscillations, the numerical coefficient would be larger than this obtained under the hypothesis of homogeneity.

The term p_0 is the dilatation and is expressed (see equation 7) in terms of the sun's mean radius as unity. At both its maximum and its minimum $K(t)=0$. An oscillation of this type of $0''.1$ in the sun's diameter means that p^0 varies over the range $\frac{1}{19,200}$, or from $-\frac{1}{38,400}$ to $+\frac{1}{38,400}$. This makes a change in T of 1400° C. That is, *a variation in the sun's apparent dimensions of $0''.1$, which is at the limit of direct observational determination, if not actually beyond it, would produce a change of its temperature of 1400° C.* This result is not so surprising when we remember how small an annual shrinkage in the sun will make up for its loss of radiant energy.

The term p_2 is the coefficient of the second order zonal harmonic. When it is positive (neglecting now all other terms) the mass is prolate, and when it is negative the mass is oblate. If it varies periodically, the mass varies from the prolate form to the oblate periodically, and equation (40) gives the temperature changes during these oscillations. Since $P_2(0)=1$ and $P_2(\frac{1}{2}\pi)\doteq-\frac{1}{2}$ the oscillations in the polar diameter are twice those of the equatorial. This deviation from sphericity enters in the second degree, and therefore a small value of it produces a much smaller change in T than that produced by the dilatational term. For example, if p_2 produces an extreme oscillation in the polar diameter of $2''.0$ as seen from the earth the change in T for the sun is only 3° C. The temperature is least when it is prolate or oblate, being equal in the two cases. The number just computed is really the upper limit of change under the hypothesis made, for at the time when $p_2=0$ the kinetic energy of oscillation is a maximum. It is impossible to compute the amount of energy which is kinetic without determining completely the circumstances of the oscillations. But

in an oscillation involving considerable density changes, where the greater part of the mass moves but slightly, it will be relatively small.

Neglecting the kinetic energy of oscillation, we find that a body having the size and mass of the sun, oscillating in such a way that the maximum value of p_2 is $\frac{1}{70}$, will undergo temperature variations of 2200° C. Its greatest oblateness is only $\frac{1}{477}$, or one-fourth that of the planet *Saturn* at the present time. This temperature change would be reduced by the fact that some of the energy would be kinetic at the time the body was spherical.

Let us now obtain some idea of the upper limit of the kinetic energy at the time the mass is spherical and passing from the oblate form to the prolate. Since kinetic energy is proportional to the product of the first power of the mass and second power of the velocity, we shall obtain the largest possible kinetic energy by supposing the change of form and dimensions has been accomplished by the motion of the smallest possible amount of matter. We shall satisfy this condition if we suppose the equatorial bulge is transferred to the poles. We shall have a distance of motion greater than the greatest possible, if we suppose each particle slides along the surface from the equator entirely to the pole. In the case of the sun the distance is 108×10^7 meters. Suppose that the whole period of oscillation is 3 hours, and consequently that the motion in one direction would be made in 90 minutes. Suppose that it is uniform. Then the velocity is 2×10^5 meters per second. Since a mass falling 425 meters under the acceleration of the earth's gravity has a velocity of 91 meters per second, this velocity in a mass of specific heat unity is equivalent to a temperature of one degree. A velocity of 2×10^5 meters per second is equivalent to a temperature of $\left(\frac{2\times10^5}{91}\right)^2=48\times10^5$ degrees. But equation (10) shows that the mass moved is only 125×10^{-6} of the whole sun. Consequently the motion is equivalent to a temperature of 600 degrees for the whole sun. This is to be subtracted from the 2,200 degrees of variation computed above, leaving a temperature change of 1,600 degrees. While these results can lay no claim to numerical precision, they are probably correct as to order of magnitude, and if so are sufficient for present purposes.

We shall now return to the effects of the oscillations upon the rate

of radiation. We shall consider first the dilatation expressed by the term p_0. From (39) we see that we must compute the ratio of

$$\left\{ T_0 + \left[\tfrac{3}{5} \frac{k^2 m}{sa} - \frac{T_0}{2} \right] p_0(t) \right\}^4 \text{ to } \left\{ T_0 + \left[\tfrac{3}{5} \frac{k^2 m}{sa} - \frac{T_0}{2} \right] p_0(t_0) \right\}^4$$

in order to obtain the percentage of change of rate of radiation. This makes it necessary to know, or to assume, the value of T_0. We shall assume that for the sun $T_0 = 6000°$. The term $\frac{1}{2} p_0 T_0$, which comes in from the change in area of the radiating surface, is, in the present case, negligible in comparison with the other term. Taking $p_0(t_1) = \frac{-1}{38{,}400}$ and $p_0(t_2) = \frac{+1}{38{,}400}$ as above, and the origin of time such that $p_0(t_0) = 0$, we find from (39) that

$$H_1 = 1.56 H_0 \text{ and } H_2 = 0.61 H_0 .$$

This means that if the sun were undergoing a dilatational oscillation of extreme range of 0″.1 in diameter, the rate of radiation at minimum radius would be 2.56 times that at maximum radius. That is, the variation would be more than one star magnitude.

The sun is certainly undergoing no changes in rate of radiation comparable to the results just found. Consequently we conclude that it cannot be undergoing dilatational oscillations capable of being detected by direct measurements of its diameter. However, Langley and Abbot thought they had observational evidence of short-period variations of about 10 per cent.[1] If we assume that the observed fluctuations are real and caused by oscillations in the sun of the type under consideration, we find from the formulae above that the apparent variations in the sun's diameter as seen from the earth are only 0″.01.

Now suppose $p_0 = 0$ and that the body is undergoing an oscillation specified by p_2. As above, we shall suppose that its maximum value is $\frac{1}{10}$. The temperature change is about 1600 degrees, and the ratio of the rate of radiation at maximum to that at minimum is found to be 2.6, or slightly more than a star magnitude. To account for a variation in radiation of 10 per cent. we should have to give p_2 the value 0.0039, which in the sun would amount to a periodic oscillation in the equatorial diameter, as seen from the earth, of 7″.4.

[1] *Astrophysical Journal*, **19**, 305, 1904.

VIII. APPLICATIONS TO VARIABLE STARS[1]

The telescope and the spectroscope prove that binary stars are very numerous. The planes of the orbits of a certain fraction of them lie so that, as seen from the earth, the two members periodically mutually eclipse each other, at least partially. Those stars in which the duration of obscuration is only a small fraction of the period of revolution are known as the *Algol* variables.

Observations of the *Algol* variables have revealed a number of important facts. These stars are all of the first spectral type. The relations between their periods and their durations of obscuration show that they are very tenuous. A remarkable fact connected with them is that in many cases their periods slowly oscillate.

There is another class of variable stars, of which δ *Cephei* is the type, having the following peculiarities: their spectra are of the solar type; their brightness changes continuously; the intervals from their maxima to their minima are much longer than those from their minima to their maxima; their spectral lines shift in their light-periods, but with such phase that they cannot be eclipsing variables; besides the chief maxima and minima they have secondary maxima and minima which are often pronounced; their periods are short (generally 3 to 10 days); and their maxima and minima are not always the same.

The variables of the β *Lyrae* type constitute another important class. Their brightness changes continuously, the maxima being divided nearly symmetrically by two unequal minima. The lines in the spectrum of β *Lyrae* shift regularly in the same period as the light-variations, but the significance of the changes is at present not certainly understood.

We shall now raise the question whether oscillations in stars of the type considered above may not be factors in their variability. That oscillations sometimes exist is not improbable. If a star suffers a violent disturbance, such as a collision with another body of plane-

[1] Ritter made applications of his results to variable stars, *op. cit.*, **8**, § 26, 179 (1879), and **13**, § 42, 366 (1881). As has been stated, he limited his discussion to the dilatational oscillations, which are much less apt to be produced by disturbances than those specified by harmonics of the second and third orders, etc. Moreover, in their quantitative effects, the dilatational oscillations stand in a class by themselves, and the results regarding them alone might easily be misleading.

tary dimensions or larger, oscillations will be set up and will persist for a very long time. If in the birth of a binary star there are violent changes in dimensions or shape, oscillations will be produced. In fact, two stars of a binary system revolving near each other in an eccentric orbit will disturb each other in such a way as to produce oscillations. The disturbing potential is a sum of zonal harmonics, $P_2, P_3, \ldots$, like those considered in equation (7). Consequently we may reasonably conclude that close binaries may be undergoing oscillations, and it is not unreasonable to suppose that single stars sometimes do also.

Consider an eclipse variable undergoing slight oscillations. The periods of the vibrations will depend primarily upon the density of the mass and the orders of the harmonics describing them. They will produce alternate maxima and minima with continuously changing light. These light-variations will compound with the apparent changes produced by the eclipses. If the former are relatively small they will introduce slight modifications of the eclipse light-curve. Since the periods of the oscillations will in general be quite different from that of revolution, the phases will progressively change relatively to each other. This will produce periodically changing maxima and minima, and will change the period from minimum to minimum as they move along the slopes of the light-curve. In a word, apparently some of the peculiarities of the *Algol* variables can be explained by supposing the effects of oscillations of the type considered here supplement the eclipses.

Consider a single star undergoing large oscillations such as treated above. The radiation will vary continuously. The maxima of light, occurring when it is spherical, will all be the same. (This would not be strictly true if the coefficients of the harmonics $P_3, P_4, \ldots$ were relatively sensible.) Considering the term depending on P_2 alone, the alternate minima will be more or less unequal; one will occur when the body is prolate with the expansion at the ends of an axis, and the other when it is oblate with the expansion along a great circle. They will be different both because of the difference in the shape of the body at the two times, and also because of its unsymmetrical orientation relatively to the earth. There will undoubtedly also be oscillations specified by harmonics of higher order, but they

will have been more rapidly destroyed by viscosity of the mass and hence, in general, will be of secondary importance. Consequently if the magnitude of a star were made to vary by this cause alone, its chief maxima would all be about the same, it might have two chief minima, and there would be secondary maxima and minima depending upon the higher-order oscillations. The spectral lines might be periodically modified and shifted by the pressure changes.

We see from equation (38) that for an oscillation of a given fraction of a star's radius, the temperature changes are directly proportional to its mass and inversely proportional to its mean radius. Or, the temperature change is proportional to the product of the density and the square of the radius. It follows from (39) that the higher the mean temperature the less the change in radiation for a given change in temperature. Equation (4) enables us to get an idea of the order of density for an oscillation of given period. Variable stars of the δ *Cephei* type have periods of from less than one day (*R Muscae*, 21^h) to nearly 30 days (*T Monocerotis*, 27^d). Most of them are considerably under 10 days. Let us compute the density from (4) for an oscillation of type P_2 having a period of 5 days. We easily find $\sigma = 10^{-3}$ on the water standard. This is small, but the density would come out less than three times as great if we attempted to explain the variation by supposing that two stars revolving essentially in contact eclipse each other. The same ratio of densities by the two hypotheses is maintained for all periods, both depending upon the inverse square of the period. For example, the density of β *Lyrae*, period 12.9 days, comes out about 14×10^{-5} that of water, while Myers found under the eclipse theory a mean density for it less than that of our atmosphere at sea-level.[1]

If the densities obtained are suspiciously small, the eclipse hypothesis is slightly preferable to the oscillation hypothesis; but in most other respects, such as the irregularities in many of the light-curves, the secondary maxima, varying maxima and minima, varying periods, and perhaps in spectroscopic behavior, the latter seems less incredible. It has been shown above that light-variations of the observed range might be produced by very moderate oscillations. If we are entitled to any conclusion at all in the matter, it is that oscillations are prob-

[1] *Astrophysical Journal*, **7**, 1, 1897.

ably sensible factors in the light-variations in close binaries, and that they may possibly be the sole cause of variation in some stars. We shall almost certainly be wrong if we suppose the light-variations in all variable stars are produced by any one cause.

IX. SUMMARY

The problems considered in this paper are the shape of the sun, assuming that it is in a state of equilibrium of rotation, and some of the consequences of its possible oscillations.

Under the hypothesis that the sun is rotating without oscillations, we find that it must be nearly an oblate spheroid, and that the difference between its equatorial and polar diameters must be less than 0".07 as seen from the earth.

It is well known that such a fluid mass as the sun may undergo oscillations whose amplitude and character depend upon the nature of the disturbances producing them, and whose periods depend upon the character of the oscillations, and upon its mass, density, and elasticity. Whatever the nature of the oscillations, the equation of the surface of the body can be represented as the sum of a converging series of spherical harmonics whose coefficients are periodic functions of the time. For simplicity, it is assumed that the mass always has an axis of symmetry so that its surface can be defined by zonal harmonics. It is also assumed that at every instant the mass is homogeneous throughout. With these restrictions, the surface, volume, moment of inertia, and self-potential of this mass of general variable form are computed.

The first practical question is whether a dilatational oscillation will not change the rate of rotation so that the change of period of rotation can be observed even though the change in diameter is beyond direct observation. The answer is in the negative, for a shrinkage of diameter of 1".0, as seen from the earth, would decrease the period of rotation by only 38 minutes.

The second question is regarding the change in temperature produced by a dilatational oscillation. An oscillation of this type such that the whole change in the sun's diameter as seen from earth is only 0".1 is, assuming the specific heat of the sun as unity, 1400° C.

It is found then, assuming Stefan's law of radiation, and that the

absolute temperature of the sun at mean radius is 6000°, that a variation of temperature 700° each side of this mean would result in a variation from 61 per cent. to 156 per cent. of that at mean radius. That is, the radiation at maximum would be 2.56 times that at minimum, or a variation of little more than a star magnitude. Consequently, if the sun were oscillating in this way the changes in its rate of radiation would be observed long before those in its diameter. An extreme oscillation of 0″.01 in the sun's diameter as seen from the earth would account for the variations of 10 per cent. in the solar radiation observed by Langley and Abbot.

The question of oscillations defined by zonal harmonics of the second order is considered. The potential energy is a minimum when the coefficient of this harmonic passes through zero, that is, at the instant when the body is spherical, and has two equal maxima when the body is most prolate and most oblate. But when the potential energy is a minimum the kinetic energy of the oscillation is a maximum. Consequently the range of variation of temperature cannot be exactly computed without knowing the motion of each element of mass at the critical times. Since this is an unsolved problem, a rough computation was made to determine an upper limit to the kinetic energy at the time the mass is spherical, and therefore a lower limit to the range of variation of temperature. It was found that a variation of this type such that the equatorial diameter changes by $\frac{1}{10}$ its value would cause a variation of temperature of 1600° C. At its greatest oblateness it would be relatively flattened one-fourth as much as *Saturn* is now. The radiation at maximum would be 2.6 times that at minimum, or more than a star magnitude greater. An oscillation of this type producing a change of 10 per cent. in the solar radiation would cause a variation in the sun's equatorial diameter, as seen from the earth, of 7″.4.

The period of these oscillations is of interest. Kelvin's formula shows that if the sun were a perfect incompressible fluid the period of the oscillations depending upon the harmonics of the second order would be 3 hours and 8 minutes. The period is independent of the mass and varies inversely as the square root of the density. The work of Lamb shows that if the mass had the viscosity of water the amplitude of the oscillation would be reduced to 37 per cent. of its original value

in 22×10^{14} years. The former of these two results is almost certainly of the right order of magnitude for the actual sun; the latter is probably much too great.

The results obtained above suggest the idea that certain classes of variable stars may owe their variability, at least partially, to oscillations. Reference is made particularly to stars whose light changes continually, such as those of which δ *Cephei* is the type, and those of the β *Lyrae* type. The spectroscope proves that in the δ *Cephei* stars the light-variations are not eclipse phenomena, and in case of β *Lyrae* the phenomena are so complicated as to suggest that eclipses are not the sole cause of the variability. If oscillations of sufficient magnitude are admitted they would explain rather satisfactorily most of the phenomena of these classes of stars. For a star of the mass and density of the sun an oscillation defined by the second-order harmonic amounting to $\frac{1}{50}$ of its radius would account for all the light-changes in these classes of stars. However, the periods of variability demand very low densities, yet greater than one-third of those required if we attempt to explain the light-variations on the eclipse theory. For a period of 5 days the density is 10^{-3} on the water standard.

The question at once arises as to the cause of the possible oscillations. In the case of a binary system having an eccentric orbit, the periodically changing disturbance of one star by the other is an efficient cause. A single star would be set vibrating by a collision with another body even of planetary mass, and by passing near another star. The fact that the sun now has a remarkable equatorial acceleration, as well as an unexplained 11-year cycle, tends to make us open-minded regarding the much simpler oscillations treated in this paper.

THE UNIVERSITY OF CHICAGO
January 20, 1909

IMPROVEMENTS AND NEW RESULTS IN SOLAR CONSTANT DETERMINATIONS[1]

By C. G. ABBOT and F. E. FOWLE, JR.

Our measurements of the "solar constant" of radiation, published as Part I of Vol. II of the *Annals of the Astrophysical Observatory of the Smithsonian Institution*, were based on bolometric observations between wave-lengths 0.387 μ and 2.428 μ of the intensity of the solar spectrum of a 60° flint-glass prism. In order to connect the bolometric intensities with a fixed scale of energy, we summed up the total area included within the spectral energy-curve, first applying certain corrections for instrumental absorption, and for the small amounts of solar radiation which reach the earth's surface, having respectively less and greater wave-lengths than those observed. These corrected areas we made proportional to pyrheliometer readings obtained simultaneously with the bolometric observations. A full description of the process employed will be found in chap. iii of Part I of the *Annals*.

INFLUENCE OF EXTRA-ATMOSPHERIC ENERGY-CURVE ON THE "SOLAR CONSTANT"

One by-product of the research was a determination of the form of the solar spectrum energy-curve on the normal or wave-length scale, as it would be outside the atmosphere. We made use of the corresponding curve on the prismatic scale in our reductions, but the discussion of sources of error made it clear that no very high degree of accuracy in the form of it was required for our purposes. In further illustration of this point we now give the results of two independent computations of the "solar constant" from observations of September 15, 1908, reduced with the two following widely different assumptions as to the form of the prismatic extra-atmospheric energy-curve. The figures give the intensities for the mean form of 1905–1906 and for the form determined from the observations of September 15, 1908.

[1] Published by permission of the secretary of the Smithsonian Institution.

Wave-Lengths	0μ.300	0.4494	0.5094	0.5525	0.6098	0.6925	0.818	1.034	1.508	2.060
1905–1906 form.	201	477	672	831	1005	1202	1396	1393	833	230
1908 form.....	102	392	564	726	861	1105	1379	1409	906	242
"Solar constant"	First result: 2.020					Second result: 2.034				

The two results differ by little more than one-half of 1 per cent., and in the opposite direction to that which should follow from the two different assumptions involved in the reduction, so that their difference is attributable to accidental changes made in the graphical and numerical reductions.

COMPUTED EFFECTIVE TEMPERATURE OF THE SUN

Not requiring, and hindered by great difficulties from reaching, high accuracy in the form of the extra-atmospheric solar energy-curve, we contented ourselves in 1905–1906 with the determinations given on p. 105 of the *Annals*. Recently Goldhammer[1] has employed our early observations at Washington and those of Langley at Mt. Whitney from which to compute the effective absolute temperature of the sun by means of the Planck formula.

$$e_\lambda \lambda^5 \left(e^{\frac{c_2}{\lambda T}} - 1\right) = c_1 .$$

At our suggestion Mr. L. B. Aldrich has employed the values given in the *Annals*[2] according to Goldhammer's process, with the following results:

λ	T	λ	T	λ	T	λ	T	λ	T
0μ.3853	5677°	0μ.4343	6279°	0μ.5226	6230°	0μ.7280	6053°	1μ.508	6242°
.3897	6125	.4417	6336	.5370	6242	.7690	6006	1.648	6558
.3942	6353	.4494	6404	.5525	6215	.818	5968	1.786	6816
.3991	6463	.4578	6366	.5697	6200	.877	5901	1.924	5823
.4042	6404	.4666	6355	.5889	6237	.946	5824	2.060	5311
.4091	6418	.4762	6354	.6098	6163	1.034	5839	2.196	4783
.4147	6300	.4861	6312	.6333	6137	1.127	5846	2.316	4550
.4210	6285	.4974	6265	.6610	6089	1.239	5953	2.428	4082
.4275	6289	.5094	6241	.6925	6073	1.367	6036		

[1] *Annalen der Physik* (4), **25**, 905–920, 1908.

[2] Change top numbers of last three columns of Table 16 of the *Annals* as follows:

6.940	798	0.3853
6.603	1326	.3897
6.279	1670	.3942
5.956	1859	.3991
5.641	1782	.4042
5.390	1817	.4091
5.048	1636	.4147

These results indicate an effective solar temperature not far from 6200° Abs.

SOLAR SPECTRUM AND SPECTRUM OF "BLACK BODY"

Mr. Aldrich computed by Planck's energy formula the "black body" curve for absolute temperature 6200° and also for 7000° and compared them with the mean observed curve as published by us in Plate XVI of the *Annals*. The intensities having been matched at wave-length 0.5697, there are found the following percentage differences at other wave-lengths:

λ	O.–C. Per Cent.		λ	O.–C. Per Cent.	
	T = 6200°	T = 7000°		T = 6200°	T = 7000°
0.40.....	+22.0	+3.0	0.80....	− 9.2	+ 1.8
.41.....	+17.0	0.0	0.90....	−18.3	+ 0.5
.43.....	+ 7.2	−8.4	1.00....	−20.7	+ 1.7
.44.....	+10.0	−2.8	1.10....	−14.7	+ 5.4
.45.....	+12.3	+0.7	1.20....	−12.9	+10.1
.47.....	+11.2	+2.8	1.30....	− 9.7	+12.4
.50.....	+ 4.3	−1.9	1.50....	+ 1.8	+28.9
.60.....	− 2.2	−0.3	1.70....	+ 7.3	+29.2
.70.....	− 7.5	+1.1	2.00....	− 2.8	+22.0

It will be seen that neither computed curve fits the observed curve closely, but except at wave-lengths greater than 1.2 μ the 7000° curve agrees better.

CORRECTIONS FOR SOLAR SPECTRUM NOT OBSERVED

In connection with these comparisons we were led to inquire whether the fractional increases of observed areas of curves, which we had allowed for solar energy of less wave-lengths than 0.387 μ and greater than 2.43 μ, were justified. We had allowed approximately as follows:

Wave-Lengths	0.000 to 0.387 μ	2.428 μ to ∞
Mt. Wilson.....	0.0040*	0.0050*
Extra-atmosphere...	0.0130	0.0055

* These two are average values, the real corrections depending on the intensity at near-by regions of the spectrum.

We are now of the opinion that the allowances are too small, especially outside the atmosphere. If the extra-atmospheric solar energy-curve is as near in form to that of the "black body" in these

unobserved regions as in the parts of the spectrum observed, we have greatly underestimated the corrections. In the last three months of the stay of the Smithsonian expedition on Mt. Wilson, in 1908, the bolometric observations were pushed to the very end of the ultra-violet spectrum transmissible by the spectroscope, namely to wave-length 0.360 μ. In all the work we have observed as far down in the infra-red as the glass prism would permit. We intend to employ in future work a quartz prism and magnalium mirrors, and hope to extend the observations from wave-length 0.29 μ to wave-length 4.0 μ.

CHANGES OF SENSITIVENESS

One of the most unsatisfactory things in our published work was the necessity stated on p. 77 of the *Annals*, of applying a correcting factor for an apparent change of sensitiveness of the bolometric apparatus. This correction was determined by comparing the corrected area of the energy-spectrum curves with the readings of the pyrheliometer. It was invariably found that the areas were smaller as compared with the pyrheliometer readings when the sun was high than when it was low. To see if this depended on the pyrheliometry, a comparison was made between two pyrheliometers, one of which pointed directly at the sun, and the other of which, pointing at a mirror, could be read either horizontal or vertical without altering the angle of incidence of the sun's rays upon the mirror. No change of sensitiveness depending on the inclination of the pyrheliometer was found, hence this comparison showed clearly that no appreciable part of the above-mentioned correction was traceable to the pyrheliometer readings. We at first thought the bolometric apparatus altered in its sensitiveness on account of change of temperature. But great successive improvements in temperature-control gave no lessening of the range of the correcting factors between 1905, 1906, and 1908. Indeed some recent comparisons perhaps indicate that the range of sensitiveness has increased in these successive years. We thought it possible that the cause of the change might be the diurnal variation of the earth's magnetism. In September 1908 a great change in the system of magnetic control of the galvanometer was made, including a very careful astaticizing of the needle system, so that the earth's magnetic field thereafter played a much subordinated part. We have

not yet reduced enough of the work of October 1908 to be sure whether this diminished the range of sensitiveness. We now think it possible that the deficient allowance heretofore made for ultra-violet radiation observed by the pyrheliometer but not by the bolometer may account for a part of the troublesome apparent change of sensitiveness.

CORRECTION OF GALVANOMETER SCALE

In studying the possible causes of the effect just discussed it was noted that the correcting factors in 1908 showed a marked stairlike succession. That is, the sensitiveness of the bolometric apparatus seemed to increase fast from the first observation to the second, then nearly halt from second to third, increase fast from third to fourth, and so on. This indicated a non-uniformity of the galvanometer scale. For the second bolographic record was always started at about 1½ cm to the south of the first; after it the plate was reversed, and the third and fourth records were placed like the first and second. By trial it was found in August 1908 that the galvanometer was in fact somewhat more sensitive for deflections on the north end of the scale. In September this defect was twice entirely removed; first, accidently, by raising the time of swing, and to that end rearranging the two control magnets which were above and below the galvanometer; second, designedly, after many experiments, by reastaticizing the needle system, removing the lower control magnet altogether, and rotating the plane of the needle system and coils from the true north-and-south plane, which they had always before occupied, to the magnetic north-and-south plane, which they will hereafter always occupy, and have occupied since September 20, 1908. Since that time there has been no scale-error great enough to be discovered by the most careful search over the whole 20 cm long scale of the galvanometer, or indeed for 20 cm on the negative side of its zero.

NEW AND ALTERED EXTRA-ATMOSPHERIC SOLAR ENERGY-CURVE

In September and October 1908 with these favorable scale conditions six days were devoted to complete independent determinations of all the elements of a "solar constant" measurement. When the results of this work were reduced we were shocked to find that the form of the extra-atmospheric solar enegy-curve differed very greatly

from the mean result of 1905 and 1906, which we had used as the basis of a large number of computations of the "solar constant" of radiation. It has been our general practice heretofore to assume that the work of the year 1905 sufficed to determine accurately enough for our purposes the form of the extra-atmospheric solar energy-curve, and that there is probably no great change in it from year to year. Hence we have seldom determined the reflecting power of the coelostat and the transmission of the spectroscope by direct observation, but have generally computed the transmission of the whole apparatus from the known form of the energy-curve outside the atmosphere, combined with the bolographic work of the given day. Fortunately, as already demonstrated above, even the wide difference between the extra-atmospheric curve of September 15, 1908, and the mean curve of 1905–1906 produces no change in the computed value of the "solar constant." This may seem at first surprising, but the reader must remember that a value within a few per cent. of the true "solar constant" can be obtained from pyrheliometry alone, as was done by Pouillet, with no knowledge whatever of the distribution of solar energy in the spectrum.

At our request Miss Graves has compared all the elements of the determination of September 17, 1908 (which is near the mean of recent results), with corresponding elements of the determination of August 7, 1906. In order to make a fair comparison we have selected original bolographic records taken with about the same zenith distance of the sun. The air-masses range from 1.292 to 1.262 for the earlier day,

Column 1	2	3	4	5	6
0$^{\mu}$.4091	0.751	1.030	1.017	1.024	0.830
.4417	.825	1.048	1.063	1.038	0.866
.4861	.886	1.074	1.016	1.060	0.977
.5525	.891	1.056	0.986	1.049	1.004
.6610	.977	1.011	1.016	0.994	1.037
.877	1.031	1.006	1.077	0.982	1.147
1.367	1.041	1.037	1.107	1.013	1.230
2.060	1.026	1.053	1.176	1.042	1.297

and from 1.268 to 1.252 for the latter. The columns of the above table give (1) wave-lengths, (2)–(6) ratios, $\frac{\text{September 17, 1908}}{\text{August 7, 1906}}$: (2) observed prismatic intensities; (3) multipliers for correcting for

coelostat absorption; (4) multipliers for correcting for spectroscope absorption; (5) atmospheric transmission coefficients; and (6) extra-atmospheric prismatic intensities.

APPARENT EXTRAORDINARY WEAKNESS OF VIOLET IN 1908

The table shows plainly that, excepting the last somewhat doubtful point where glass absorption and water-vapor absorption are both large, the transmission of the atmosphere and of the apparatus was almost alike on the two days, *but the original energy-curve* and hence the extra-atmospheric curve were far higher in the violet in August 1906 than in September 1908. This result, if true, indicates an enormous decrease in the sun's violet-emission. But the "solar constants" for the two days indicate no change in the sun's total emission. They are: August 7, 1906: 2.033; September 17, 1908: 2.046.

We now regret that we have not complete independent determinations of the transmission of the apparatus for all parts of the years 1906 and 1908. However, taking days when the mirrors had been freshly silvered, or freshly polished, it is to be supposed that the transmission would then be as good as on the days just compared. Two such days are June 21 and September 2, 1908, and on the latter the transmission of the apparatus was probably even better than on September 17, 1908. We now give the ratio of intensities of original bolographic curves taken at about the same air-masses for these three days of 1908 as compared with the curve of August 7, 1906. The atmospheric transmission was almost exactly the same on all these days.

Wave-Length in μ		0.4091	0.4417	0.4861	0.5525	0.6610	0.877	1.367	2.060
Intensity Ratio	June 21.....	0.888	0.928	0.941	0.984	1.011	1.040	1.022	1.136
	Sept. 2.....	0.867	0.935	0.918	0.935	0.977	1.013	0.998	1.045
	Sept. 17.....	0.830	0.866	0.977	1.004	1.037	1.147	1.230	1.297

All three days indicate the same result, namely: in 1908 the intensity of violet rays in sunlight was less as compared with red rays than in August 1906. The reader may see by inspection of Table 16 of Volume II of the *Annals* that in August 1906 the violet was apparently weaker as compared with the red than on the average of days in 1905.

Either these results are affected by some unknown experimental error or the distribution of energy in the solar spectrum is subject to great secular changes without great attending changes of the solar constant. We shall test the matter in future work; and with the new quartz-and-magnalium outfit in addition to the present glass-and-silver one, instrumental errors can hardly remain a possible explanation if such changes are in future observed.

A NEW STANDARD PYRHELIOMETER

As stated in the *Annals*, all our "solar constant" results are given in terms of a scale of energy whose unit is probably a few per cent. smaller than the gram-calory or therm. We have made many efforts to determine the ratio of our unit to the therm, and have designed and tried several new supposedly standard pyrheliometers, in which the hollow chamber or "black body" feature was introduced, and the solar heating was determined by measuring the rise of temperature of a current of water. Known electrical heating could also be measured in the same way as a check. Not satisfied with the results obtained with the instrument of this kind employed in 1905–1906, a new one of improved construction and altered size was used in 1908. This behaved in an exemplary manner, and many comparisons were made between it and the secondary pyrheliometers IV and VII used in daily "solar constant" measurements. Also many check measurements were made by means of two electrical heating coils. The results are so concordant throughout, and the electrical heat introduced was so exactly found in the water-flow in every instance, that we now believe the standard scale is established. Nevertheless our 1908 results indicate that the readings of the supposed standard of 1906 published at p. 45 of the *Annals* are 5.9 per cent. too high, and all the pyrheliometer and "solar constant" results published in the *Annals*, if reduced to our new provisional scale of real calories, would be 7.6 per cent. lower. Kimball[1] has found our arbitrary scale to be 11.0 per cent. above that of new Ångström pyrheliometers. Our new provisional standard scale is then about 2.6 per cent. above Ångström's.

[1] *Bulletin Mt. Weather Observatory*, Vol. I, Part 2.

"SOLAR CONSTANT" CORRECTIONS

We now suppose that our "solar constant" values of 1902–1906 must be lowered 7.6 per cent. on account of pyrheliometry, and raised perhaps 10 per cent. on account of deficient correction for ultra-violet and infra-red unobserved energy. With our new quartz-magnalium spectrograph and a projected new pyrheliometer we hope to assure ourselves of these points in this present year, 1909.

ASTROPHYSICAL OBSERVATORY
SMITHSONIAN INSTITUTION
April 1909

CAMERA OBJECTIVES FOR SPECTROGRAPHS

By J. S. PLASKETT

It is well known that the camera objectives in general use in stellar spectrographic work have a very limited field of good definition, not exceeding in general 2°, which covers, in the usual dispersion of three prisms, about 200 tenth-meters. While this is a sufficient range for spectra of the second type, which are rich in lines, it is not sufficient for early-type spectra which may contain only one or two lines in this region and in which, consequently, the errors of measurement will be high. As practically the whole photographic region of the spectrum, $H\beta$ to K, may be obtained in one exposure with either refractor or reflector, it is evident that a considerable gain in the measurable material in such spectra would be obtained, without increase in exposure time, if a camera lens giving a considerably wider field were available.

Various attempts at the solution of this problem have been made, of which the most successful known to me is that described by Hartmann.[1] This objective made by Zeiss, known as the "Chromat," is constructed of the same material as the prisms, and is composed of two simple meniscus elements, one positive, one negative, separated by a small air space. As there is no chromatic correction the spectra are brought into focus by inclining the plates toward the violet, about 16° from the normal to the axis with a dispersion of three prisms. According to Hartmann this objective gives a flat field of 14°. A Zeiss "Chromat" has been in use in Ottawa for considerably over a year, entirely fulfilling expectations and giving, after slightly increasing the separation of the elements, the whole field used, from $H\beta$ to $H\delta$ (about 8°), almost absolutely flat with excellent definition. There can be no doubt that the field would extend farther if necessary.

Unfortunately, as was learned upon inquiry from Zeiss, this type of objective cannot be successfully made of a larger angular aperture than about *f.* 12. This was confirmed by the performance of a shorter focus lens of the same type (aperture ratio *f.* 8) constructed by Brashear which gave inferior definition. More recently, however,

[1] *Zeitschrift für Instrumentenkunde*, **24**, 257, 1904.

Ross Limited, London, have designed and constructed especially to conform to our requirements a lens similar in form to their "Homocentric," consisting of four separated elements, but following the principle of the "Chromat" in being entirely made of the prism material and consequently requiring inclination of the plate to bring the spectrum into focus. This lens, which will be more fully discussed later, gives, at an aperture ratio of *f.* 5.6, excellent definition and a flat field. The above remarks apply to a dispersion of three prisms, for which this type is especially adapted. If they were used with one prism, in addition to the limitations as to aperture, the plate inclination required (about 50°) would be inconvenient and practically inadmissible in radial velocity work.

There are consequently required short-focus objectives giving a flat field with three prisms, and objectives giving a flatter field than the regular triplet with a single prism. This need, together with what had already been accomplished by Hartmann and Zeiss, was laid before the J. A. Brashear Co. who, with their usual willingness, put their best efforts at our disposal and, in collaboration with Professor Hastings, produced two eminently successful objectives. In both of these the employment of one kind of glass only is followed, although not, as in the "Chromat," of the same material as the prisms, and the consequent chromatic differences in focus are overcome by inclining the plate. The objective first produced, to which they have given the name "Single Material," is composed of two widely separated positive elements of crown glass of the lowest dispersion and is especially adapted for use with one prism, giving exquisite definition and a field flat within 0.1 mm over the whole visible and considerably into the ultra-violet spectrum. The other is similar to the "Chromat" in form but made of light crown glass, giving also a flat field and good definition with three prisms with a plate inclination of only slightly over half that of the "Chromat."

The limiting aperture ratio of the former of these objectives is about *f.* 8, of the latter *f.* 12, so that evidently they cannot supply the need of short-focus lenses of *f.* 6 or thereabouts for either single or three-prism work.

The only prospect of success in this respect seemed to lie in some of the modern anastigmat photographic lenses, and a number of dif-

ferent makes were accordingly obtained for trial. The definition of several of these, though good enough for ordinary photographic work, would not stand the critical test of spectrum photography, owing probably to some residual spherical aberration. Two, however, the Ross "Homocentric" and the Zeiss "Tessar," gave good definition and the forms of their fields were accordingly determined.

It may not be out of place to give a brief description of the method employed in determining the fields of the ten lenses tested. The dispersion for eight of them was produced by the Ottawa spectrograph having the following optical constants: Hastings "Isokumatic" collimator objective of 35 mm aperture and 525 mm focus; one or three prisms of Jena glass O.102, angles 63° 50′ each; ray at minimum deviation λ 4415. Two of the objectives were tested with a new single-prism spectrograph having "Isokumatic" collimator of 51 mm aperture and 763 mm focus, O.102 prism of angle 63° 30′, ray at minimum λ 4325.

The positions of focus in different parts of the field were determined by a modification of Hartmann's method[1] of extra-focal exposures. By means of a revolvable semicircular diaphragm behind the collimator lens and an occulting diaphragm in front of the slit, a narrow strip of spectrum, photographed through the half of the prisms near the refracting edge, was placed between and touching two narrow strips made through the base half of the prisms. Evidently, when the plate is in the focus of the camera lens for any particular line in the spectrum, the adjacent portions of this line will be continuous, while, if not in focus, the central section will be displaced to red or violet of the outside sections, the direction and magnitude of this displacement giving a measure of the position of the focal point for the line in question. Two such plates, one inside and one outside the focus, will suffice to determine the form of the field. In order to avoid the labor of measurement and computation and on account of the diffuseness of the lines and consequent inaccuracy of measurement, when the plate is more than a millimeter from the focus, I have generally preferred to make a number of spectra, by the method outlined above, at camera settings about 0.25 mm apart within and without the focus. Five of these have in general sufficed to deter-

[1] *Zeitschrift für Instrumentenkunde*, **24**, 1, 1904.

mine the focal curve and, as the camera back can be moved laterally, they can all be made on one plate, thus allowing ready comparisons. Simple inspection of these spectra under a microscope or even by a hand magnifier enables the focus of any line to be determined to about 0.05 mm by observing at which of two successive spectra the central section has opposite displacements with respect to the outside sections. Interpolation to the above accuracy can then generally be made. This takes only one-tenth the time and is probably equally as accurate as the method of measuring the displacements and computing the distance from focus. I may say that the camera setting in our regular work is always determined in this way, enabling the plate to be certainly placed considerably within 0.1 mm of the true focus.

This method is probably open to the objection that it will not give the true focal point when the system has aberration, but it must be remembered that, to prevent systematic displacements in radial velocity work due to non-uniform illumination of the collimator objective, this method, which determines the focus by the absence of such displacement, is certainly the one that should be used. Moreover in this case tests at full aperture, so far as the focus can be determined by definition, confirmed the results of the former method and there is no reason to doubt the accuracy of the focal curves determined.

The following ten lenses, given in the order of procuring and testing, were tested for their curvature of field.

OBJECTIVES TESTED FOR FIELD

Number	Objective	Aperture	Focal Length	Tested with Dispersion of
1.....	Brashear Single Material	45	525	1 Prism, 3 Prisms
2.....	Zeiss Chromat	45	525	3 Prisms
3.....	Ross Homocentric	40	254	3 Prisms
4.....	Zeiss Tessar	48	305	3 Prisms
5.....	Brashear Light Crown	45	525	3 Prisms
6.....	Brashear Telescope Flint	45	525	3 Prisms
7.....	Brashear O.102 ("Chromat")	45	375	3 Prisms
8.....	Brashear Triplet	57	480	New 1 Prism 3 Prisms
9.....	Brashear Single Material	57	457	New 1 Prism
10.....	Ross Special Homocentric	40	254	3 Prisms

The form of field of each of the lenses is given in the accompanying figures where the horizontal lines represent differences of focus of

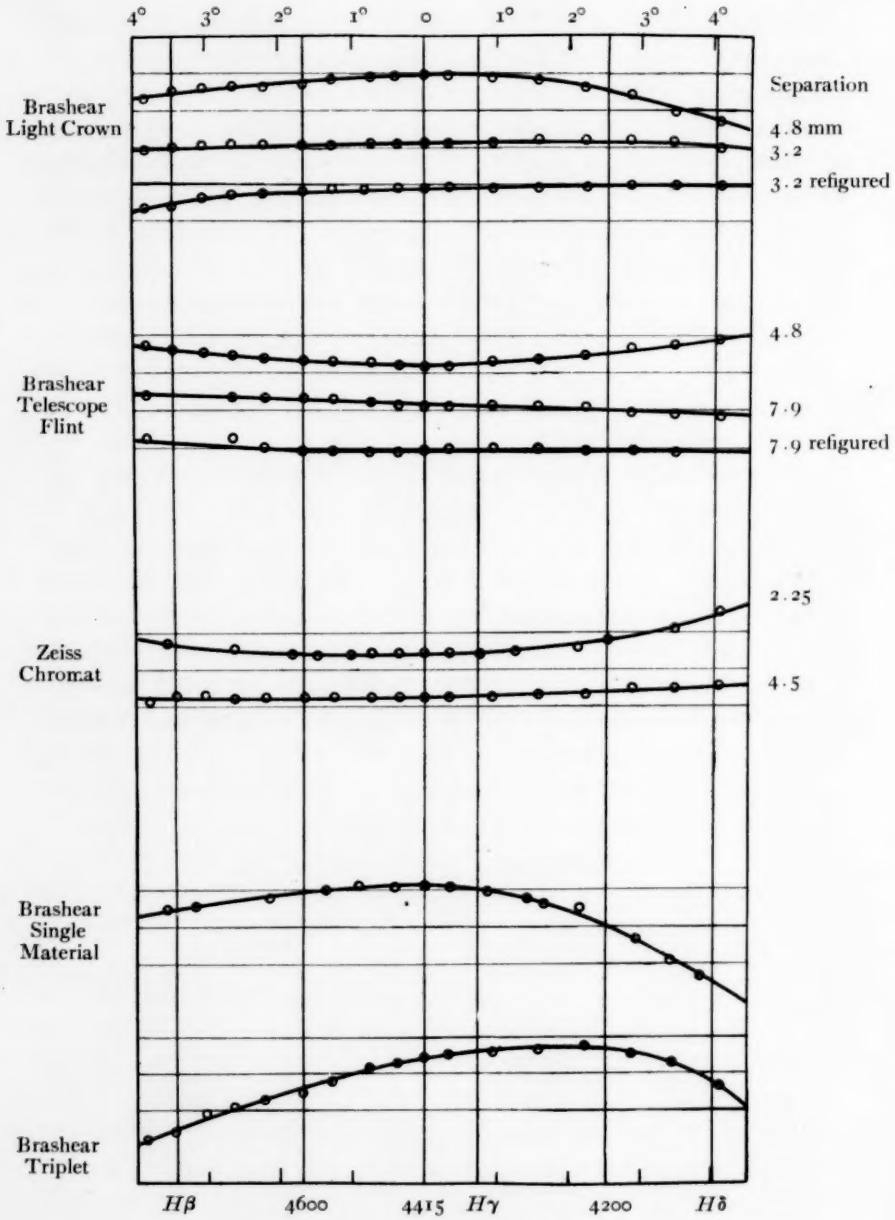


FIG. 1.—Long-focus objectives with three prisms.

one millimeter, where the wave-length and angular distance from the optical axis are shown by the vertical lines, and the diameters of the circles representing the observed points are 0.2 mm. Wherever the curves are not horizontal indicates that the inclination of the plate-holder required changing slightly, but this of course has no effect on the form of the field. In order to group the curves according to the type and purpose of the objectives, the order given in the above table has been changed and those of longer focus used with a dispersion of three prisms will be first considered (Fig. 1).

BRASHEAR SINGLE MATERIAL (NO. 1)

This consists of two simple converging lenses, the front double convex, the rear convex meniscus, of crown glass of lowest index, separated by nearly one-third the focal length. As will be seen later, this objective gives a beautiful field with one prism but is strongly concave toward the lens with three prisms, with about the same curvature of field as the regular Hastings Triplet. The inclination of the plate toward the violet is slightly over 5°. Allowing deviation from focus of 0.1 mm, slightly over 2° of field is usable.

ZEISS CHROMAT (NO. 2)

This consists of two strongly curved meniscus elements of Jena Glass O.102, the front diverging, the rear converging and of about half the focus of the combination. When received it gave a field convex toward the lens, as shown in the upper curve. When the separation between the elements was increased from 2.25 to 4.5 mm, the field became almost absolutely flat over the whole 8°, giving at the same time excellent definition. Inclination of the plate toward the violet about 16°.

BRASHEAR LIGHT CROWN (NO. 5)

This is a lens of the same form as the "Chromat" except that it is made of light crown glass. With the original separation the field was concave but became flat on decreasing the separation from 4.8 to 3.2 mm. This change in separation resulted in loss of defining power. The objective was refigured at the new separation and gave good definition and field flat over practically the whole range. Inclination of plate to the violet about 9°.

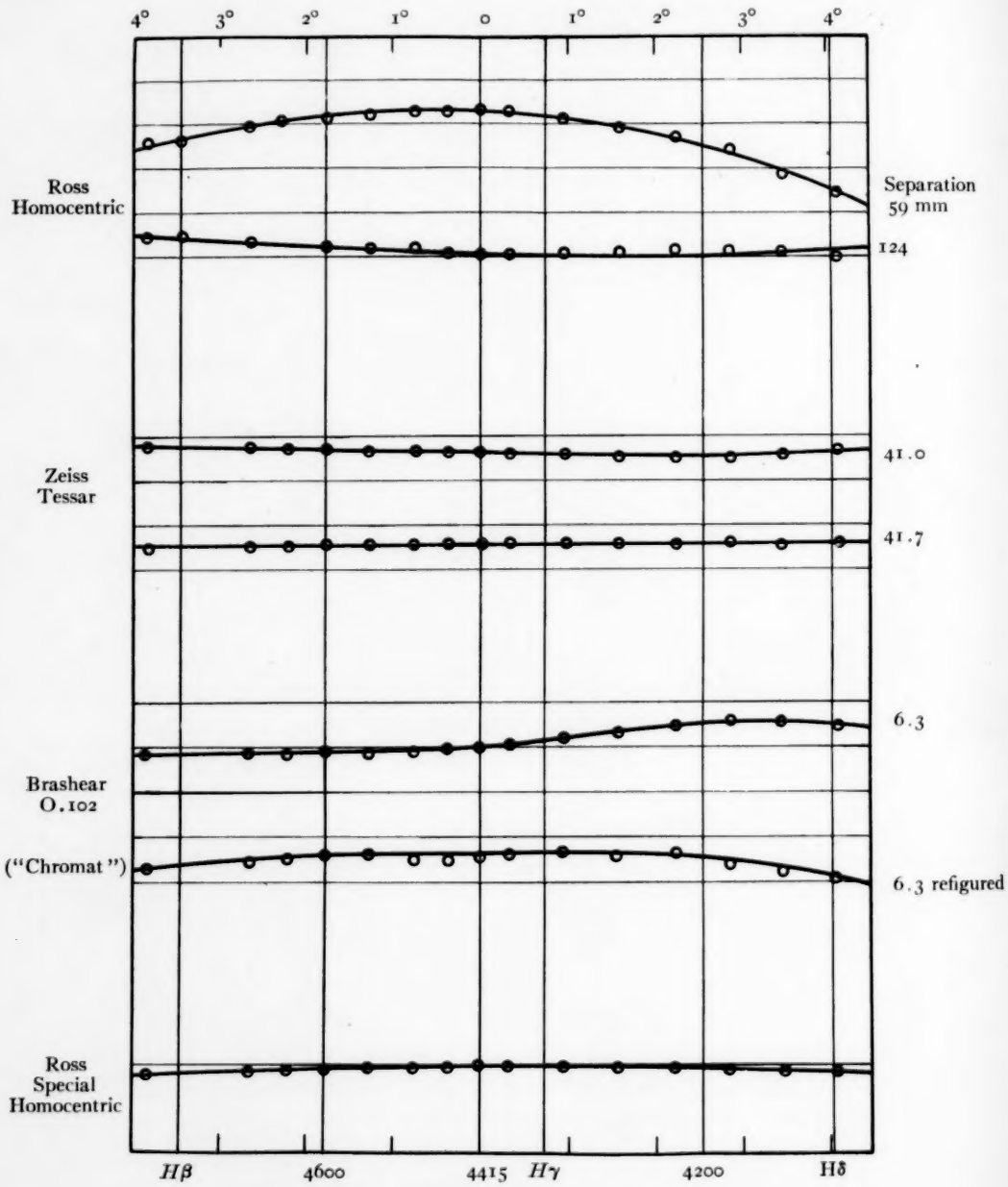


FIG. 2.—Short-focus objectives with three prisms.

BRASHEAR TELESCOPE FLINT (NO. 6)

An objective similar to the previous one only made of telescope flint glass. The field was originally convex but became flat on increasing the separation from 4.8 to 7.9 mm. Refiguring did not give so much improvement as in the light crown objective. Field is now practically flat. Inclination of plate to the violet about 13°.5.

BRASHEAR TRIPLET (HASTINGS) (NO. 8)

This is a lens of the same type as used in the Mills, Bruce, and Lowell spectrographs. The field is, as shown, strongly concave toward the objective with a usable portion, allowing deviation of 0.1 mm, of about 2°.5. The definition at the center of the field is about the same as in the "Chromat," but toward the margins even when in focus is much inferior.

All the above objectives are of relatively long focus, small angular aperture, about *f.* 12, tested with dispersion of three prisms. Let us now examine the fields given by shorter focus objectives, using the same dispersion (Fig. 2).

ROSS HOMOCENTRIC (NO. 3)

This standard photographic objective gives good definition but a strongly concave field. An increase in separation from 59 to 124 mm appears to flatten the field but at the expense of defining power and the lens is not usable at the increased separation. Useful field is not more than 2°.

ZEISS TESSAR (NO. 4)

This objective was one of the standard form taken from the stock of Bausch & Lomb. It gives good definition and a field very slightly convex. This convexity is removed by an increase in separation from 41.0 to 41.7 mm, but with a slight loss in defining power, so that it is probably preferable to use it at the normal separation. Another lens of the same series, aperture, and focus was tested, giving practically the same field but considerably poorer definition. This is of interest as showing the differences between the performance of two objectives presumably identical and indicates the desirability of specially selecting and testing the lens to be used.

BRASHEAR 0.102 ("CHROMAT") (NO. 7)

This objective, of the same type and material as the Zeiss Chromat but of larger angular aperture, gives a field nearly flat with a separation of 6.3 mm but with poor definition even after refiguring. This shows that this type cannot be successfully constructed of larger aperture ratio than *f.* 11, say. Inclination of the plate to the violet about 16°.

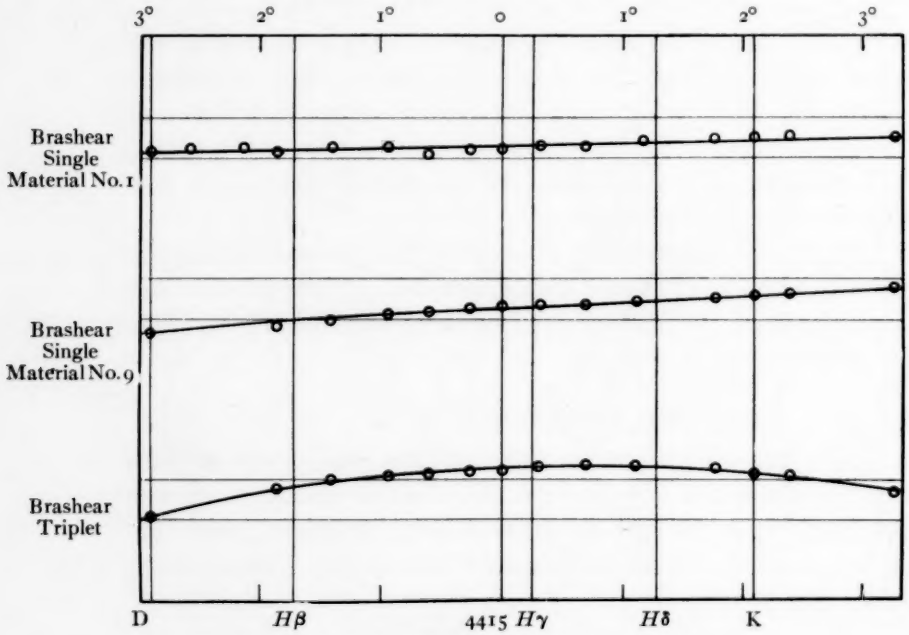


FIG. 3.—Objectives tested with one prism.

ROSS SPECIAL HOMOCENTRIC (NO. 10)

This objective was, by the kindness of the makers, Messrs. Ross Limited, especially computed and constructed for us. It has an aperture ratio of *f.* 5.6, is of practically the same form as their Homocentric but with all four elements of O.102 glass. It gives beautiful definition and a field nearly flat, usable over 8°. Change of separation is without appreciable effect on the form of field. Inclination of plate to the violet about 16°.

Two types of objectives of medium and long focus have been tested with a dispersion of one prism (Fig. 3).

BRASHEAR SINGLE MATERIAL (NOS. 1 AND 9)

Both of these objectives, whose form was described above, are of the same type, No. 1 of an aperture ratio *f.* 11.5; No. 9 of *f.* 8. When used with three prisms they give the strongly concave field shown in Fig. 1, but with a single prism the field is almost absolutely flat over the whole range of visible and as far into the ultra-violet spectrum as the prism will transmit. The definition given is excellent and the objectives leave nothing to be desired for single-prism work. The inclination of plate to the violet is about 16°. I am glad to express here my appreciation of the efforts as well as my admiration for the skill of Mr. McDowell in figuring these objectives. As both components are converging, the only means of removing the positive spherical aberration is by departure from spherical surfaces. While, as Mr. McDowell says, this was comparatively easy for No. 1, of the smaller angular aperture, it taxed even his skill to remove it entirely in the other, and it was only after a second trial and the use of a special device that the objective was finally made perfect.

BRASHEAR TRIPLET (NO. 8)

This, as with three prisms, gives a field concave toward the objective but with considerably less curvature. The usable field is somewhat over 2°. Definition good.

The final results of the investigation may be summarized as follows:

For a dispersion of three prisms with a camera of fairly long focus two objectives are much superior to the others, the Zeiss "Chromat" and the Brashear Light Crown. The former gives a flatter field and slightly better definition than the latter, but on the other hand the smaller plate inclination of 8° instead of 16° and the smaller absorption of the Brashear are an advantage. The definition of either of these is fully equal to the regular triplet in the center of the field and much superior at the margins.

For short-focus lenses with three prisms both the Zeiss Tessar and the Ross Special Homocentric give good definition and flat fields. The Ross can be used of shorter focus than the Tessar, and gives exquisite definition, but the field of the Tessar is flatter and the plate is normal to the axis.

In single-prism work the Brashear Single Material is much supe-

rior to the type of Triplet usually employed, both in definition and extent of field, and cannot be surpassed or even equaled for its purpose.

In conclusion, I wish to express my indebtedness to Dr. King, director of the Observatory, not only for the interest he has shown in this work, but also for his willingness to meet the expense of the various objectives obtained.

DOMINION OBSERVATORY, OTTAWA
March 1909

NOTE ON THE POLARIZING EFFECT OF COELOSTAT MIRRORS[1]

By CHARLES E. ST. JOHN

The discovery of the Zeeman effect in sun-spots by Hale[2] made it important to determine the action of the silver-on-glass mirrors of the tower telescope of the Mount Wilson Solar Observatory upon circularly polarized light. It was at once recognized that circularly polarized light would be changed to elliptically polarized by reflection from the silver surfaces and that its effect would vary with the angles of incidence and hence with the position of the sun. A description of the tower telescope has been given by Hale[3] and it only needs be said here that the second mirror sends the light vertically downward through the 12-inch objective and that about 4 feet below the objective—focal length 60 feet—the beam was received upon the analyzer which served to fix the position of the axes of the elliptically polarized light and to determine their ratio.

Light circularly polarized in a known direction was obtained by passing sunlight through a Nicol prism and then through a Fresnel rhomb whose plane of incidence formed an angle of 45° with the short diagonal of the Nicol. The direction in which it was necessary to rotate the rhomb to bring its plane of incidence into coincidence with the plane of vibration of the Nicol was taken as the direction of revolution of the circularly polarized light[4] and it was distinguished as clockwise and counter-clockwise as viewed by the observer when looking in the direction of the source. This polarizer was mounted in a screen and interposed between the sun and the first mirror so that no light reached the mirror except that passing through the polarizer. The analyzer consisted of a Fresnel rhomb mounted upon a divided circle, and a Nicol likewise with a graduated mounting that

[1] *Contributions from the Mount Wilson Solar Observatory*, No. 37.

[2] *Contributions from the Mount Wilson Solar Observatory*, No. 30; *Astrophysical Journal*, **28**, 315, 1908.

[3] *Contributions from the Mount Wilson Solar Observatory*, No. 23; *Astrophysical Journal*, **27**, 204, 1908.

[4] Wood, *Physical Optics*, p. 277.

could follow the eccentric movements of the rhomb as the latter was rotated. Monochromatic light was obtained by interposing ruby glass in the train and its intensity was further reduced by means of a smoked glass. The observer lay on his back while making the observations which were carried out in the following way. The incident light was polarized in a clockwise direction and after reflection from the two mirrors fell upon the rhomb; a position of the rhomb was then found for which the Nicol could produce extinction of the light; the rhomb was rotated 90° and the position of the Nicol for extinction again determined; the operation was repeated with reversed polarization of the incident light. The readings of the scales give the azimuth of the plane of incidence of the rhomb and the corresponding azimuth of the long diagonal of the Nicol for extinction. The difference between the two azimuths of the Nicol corresponding to the first and second settings of the rhomb gives twice the angle between the major axis of the elliptically polarized light and the plane polarized light produced by the rhomb when its plane of incidence coincides with the major axis of the ellipse. This angle is given in the column of the table marked I. The ratio of the axes is of course given by the tangent of this angle. The difference between positions of the Nicol for clockwise and counter-clockwise light for the same setting of the rhomb is given in the column marked II, and approximates 90°.

The great change in the position of the axis of the ellipse between morning and afternoon, as shown by the position of the rhomb, followed the transfer of the coelostat mirror from the west to the east side of the second mirror at noon. The condition of the silver surfaces affects the results appreciably, but under normal working conditions of the surfaces the changes due to this cause were not great enough to be troublesome.

In the practical application of these results in studying the Zeeman effect along the lines of force in the magnetic field of sun-spots, the rhomb was set with its plane lying northeast and southwest in azimuth 20° for the forenoon, in azimuth 70° from noon to about 3 P. M., and then in azimuth 55°. The Nicol was set with its long diagonal in an azimuth equal to that of the rhomb plus 25° in the forenoon and plus 30° in the afternoon to extinguish clockwise light, and rotated

	Forenoon					Afternoon				
		Rhomb	Nicol	I	II		Rhomb	Nicol	I	II
Clockwise	Sept. 27	23° E of N	50° E of N							
Clockwise	7:00	67 W of N	10 W of N	30°						
Clockwise	Sept. 26	25 E of N	58 E of N			Sept. 18	73° E of N	104° E of N		
Clockwise		65 W of N	0				17 W of N	52 E of N		
Counter-clockwise	8:20	25 E of N	30 W of N			1:30	73 E of N	15 E of N		
Counter-clockwise		65 W of N	87 W of N	29	87°.5		17 W of N	39 W of N	26°.5	90°
Clockwise	Sept. 26	15 E of N	48 E of N			Sept. 16	70 E of N	105 E of N		
Clockwise		75 W of N	5 W of N				20 W of N	50 E of N		
Counter-clockwise	9:35	15 E of N	42 W of N			2:00	70 E of N	17 E of N		
Counter-clockwise		75 W of N	90 W of N	25	91.5		20 W of N	40 W of N	28	89
Clockwise	Sept. 18	23 E of N	55 E of N			Sept. 25	70 E of N	105 E of N		
Clockwise		67 W of N	0				20 W of N	45 E of N		
Counter-clockwise	10:00	23 E of N	35 W of N			2:40	70 E of N	20 E of N		
Counter-clockwise		67 W of N	88 W of N	26	89		20 W of N	45 W of N	31	87.5
Clockwise	Sept. 26	12 E of N	42 E of N			Sept. 18	54 E of N	90 E of N		
Clockwise		78 W of N	8 W of N				36 W of N	25 W of N		
Counter-clockwise	10:30	15 E of N	47 W of N			3:50	54 E of N	0		
Counter-clockwise		75 W of N	93 W of N	24	87		36 W of N	60 W of N	31	87.5
Clockwise	Sept. 26	10 E of N	40 E of N			Sept. 26	63 E of N	100 E of N		
Clockwise		80 W of N	8 W of N				27 W of N	38 E of N		
Counter-clockwise	11:00	10 E of N	50 W of N			3:10	63 E of N	7 E of N		
Counter-clockwise		80 W of N	100 W of N	24.5	91		27 W of N	53 W of N	30.5	92
Clockwise	Sept. 18	23 E of N	55 E of N			Sept. 26	55 E of N	95 E of N		
Clockwise		67 W of N	2 E of N			4:05	35 W of N	30 E of N	32.5	
Counter-clockwise	11:20	23 E of N	40 W of N							
Counter-clockwise		67 W of N	88 W of N	25	92.5					
Clockwise	Sept. 16	20 E of N	47 E of N			Sept. 26	50 E of N	85 E of N		
Counter-clockwise	12:00	20 E of N	45 W of N	26		5:00	40 W of N	20 E of N	32.5	
Mean		22° E of N		26°			62° E of N		30°	

90° to extinguish counter-clockwise light. Calculation of the effect produced by the plane of the rhomb not coinciding with the major axis of the elliptically polarized light shows that for a deviation of 10° the resulting effect is very narrowly elliptical light—axis in the ratio twelve to one—with the major axis forming very nearly the same angle with the plane of the rhomb as the plane polarized light which results when the plane of the rhomb coincides with the major axis of the elliptical light coming from the mirror. This allows some latitude in the setting of the rhomb provided the angle between the planes of rhomb and Nicol is maintained.

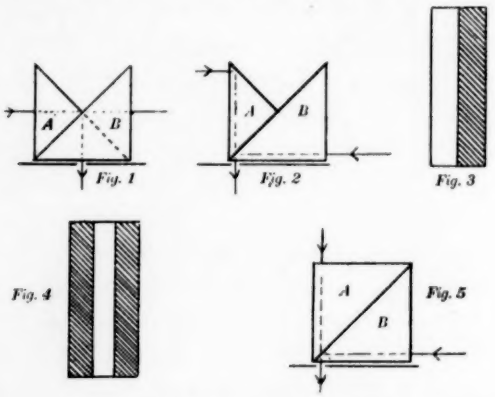
MOUNT WILSON SOLAR OBSERVATORY
February 1909

SOME CONVENIENT FORMS OF COMPARISON-PRISMS

By LOUIS BELL

Everyone who has used the spectroscope much, and especially everyone who has used forms of spectrophotometer requiring a comparison-prism over the slit, has been annoyed by the black band between the spectra produced by the edge of the comparison-prism. In spectrophotometry particularly the presence of this band is more than inconvenient. It is fatal to precision. Having recently had built a spectrophotometer of the Nichols form for use on the photometer-bar in place of the regular sight-box, the writer encountered the familiar difficulty and set about finding means of remedying it. The resulting device for overcoming the difficulty is here put on record for the benefit of others. Fig. 1 shows the familiar arrangement of prisms ordinarily used in the Nichols spectrophotometer, together with the paths of the rays from the prospective sides through the prisms and into the slit. The new device was built from this same pair of prisms. One of them, *A*, was cut down, retaining the right angle, so that its new hypothenuse dimension was equal to its old perpendicular. The other prism, *B*, was treated as follows: Its hypothenuse face was silvered densely enough to present a smooth, opaque coating. A cut through this coating centrally in the long axis of the hypothenuse face was made by a sharp ivory blade, and one-half the silver coating was completely removed, leaving the hypothenuse face half clear and half densely silvered. The arrangement of this hypothenuse face is shown in Fig. 3. Then the cut-down prism *A* was cemented with Canada balsam to the hypothenuse face of *B* as shown diagrammatically in Fig. 2. This balsam cementing should be done at a very moderate temperature lest the silver be started if the baking is too hard. In Fig. 2 the ray from the right side enters the perpendicular face of the prism *B*, just as it would in Fig. 1, but on arriving at the silvered face, that part of the beam which meets the balsamed face of clear glass passes straight through, almost unimpeded, since the balsam has very nearly the same index

of refraction as the glass of the prism. The rest of the beam is reflected at that part of the hypothenuse face which is protected from the balsam by the silvering, and passes into the slit. The ray from the other direction enters the external portion of the prism A, is reflected at its rear face, which for symmetry and the interception of stray light should also be silvered, and, in so far as the silvering permits, passes through the balsamed surface of junction of A and B and enters the slit. That part of the beam which meets the silvering is reflected out of the prism. The paths of the two rays are shown diagrammatically in Fig. 2. The line of demarkation between the two spectra provided by this comparison-prism is due to the sharp edge that limits the silver-



ing on the cemented face. In making up the prism for regular use it would probably be best to cut through the silvering with a very fine diamond point and then clear it away from this line as a boundary. The line of separation of the two rays is so sharp and its plane can be brought so close to the slit that, when the instrument is properly adjusted, the dark band usually seen with the comparison-prisms has practically disappeared entirely. In the spectrophotometer in which the experimental prism was tried, the focal length of the collimator is only 12 cm, and even under this trying condition the joining of the two spectra is very good indeed. With a collimator of such focal length as is more commonly used, even better results can be obtained. In fact with lenses of a meter or more in focus, the difference in focus for the slit and for the silvered face can be made hardly more than the difference in focus due to the secondary spectrum.

Obviously one can remove the silvering in strips as shown in Fig. 4, and in this way can get an even more delicate comparison of brightness than with the two fields given by the form shown in Fig. 3.

Where it is merely necessary to bring in a comparison-spectrum from a position at right angles to the axis of the collimator, the very obvious modification shown in Fig. 5 can be used to advantage. Here two entirely similar prisms in which the hypothenuse face of one is silvered in one or more stripes are balsamed together. The direct beam then comes straight through from the front of the slit except where it is deflected by the silvering, and the lateral beam comes in by reflection at the silvered surface just as tin the case of Fig. 2. The use of either of these forms of comparison-prisms requires careful adjustment to obtain the maximum of clearness between the contact of the two spectra, but once the adjustment is made, the result is excellent. With a collimating lens of so short focus as that employed in the spectrophotometer mentioned, it is advantageous to have screw adjustments so arranged that the lens can be slightly raised, lowered, or tipped so that line spectra thrown simultaneously in from the two directions can be matched with precision. Once this is accomplished, the adjustment can be readily preserved. The prisms in this instance were of common crown glass, but in making up special prisms for a spectrophotometer it might be desirable to pick out a crown which would match the index of refraction of the balsam with special accuracy. This would require an index of nearly 1.54, since the index of balsam rises materially as the terpenes are baked out.

BOSTON, MASS.
March 1909

THE BINARY VARIABLE STAR *RZ CASSIOPEIAE*

By JOHN STEIN

The investigations of this variable star made by J. A. Parkhurst and F. C. Jordan[1] make it of interest to see how well the light-curve, as determined by their "absolute scale" of photographic magnitudes, can be represented by the satellite theory.

In what follows I restrict myself to the simplest form of the theory. Two bodies, with radii 1 and x, respectively, the first bright, the second dark, revolve in a circular orbit of radius a. The plane of the orbit forms a small angle i with the line of sight. Consider in place of the bright star a uniformly illuminated disk of total intensity 1, in place of the satellite a dark disk; then the light-intensity J for the time t, reckoned from minimum, is given by the formulae:

$$J = 1 - \frac{M}{\pi} ;$$

$$M = \tfrac{1}{2}\{x^2 (2\phi - \sin 2\phi) + (2\phi' - \sin 2\phi')\};$$

$$\sin \phi' = x \sin \phi ; \quad \cos \phi' = \frac{\rho^2 + x^2 - 1}{2x\rho} ;$$

$$\rho^2 = a^2 \sin^2 nt + a^2 \sin^2 i \cos^2 nt .$$

In these equations M is the part of the disk of the principal star hidden by the satellite, ρ the apparent distance of their centers, ϕ' and ϕ the angles under which the common chord of the two disks is seen from the centers of the principal star and the satellite. The magnitude difference $m - m_0$ (m_0 being the normal light) is then given by

$$m - m_0 = -2.5 \log J .$$

To determine the approximate elements we have the data:

(*a*) Period of *RZ Cassiopeiae* $= 1^d.195258$.

(*b*) Normal light $6^m.43$; minimum $7^m.64$; minimum intensity $J_m = 0.3281$.

(*c*) Duration of eclipse $2t_0 = 0^d.23 = 5^h\ 32^m$.

(*d*) The minimum is sharply defined, no trace of a secondary minimum.

[1] *Astrophysical Journal*, **26**, 251–254, 1907.

Theoretical light-curves were calculated on two hypotheses:

(I) $x=1$. Then

$$M=2\phi-\sin 2\phi\ ;\quad \cos\phi=\frac{\rho}{2}\ ;$$

$$J_m=1-\frac{2\phi_m-\sin 2\phi_m}{\pi}\ ;\quad \cos\phi_m=a\sin i\ .$$

Also

$$a^2\sin^2 nt_o+a^2\cos^2 nt_o\sin^2 i=(1+x)^2.$$

Therefore we find

$$a=3.437;\quad i=8^\circ 43'\ .$$

(II) $1-x^2=J_m$; $x=0.820$. This is the minimum value of x. Since there is no stationary period at the minimum, the disks must be in internal contact when the centers are at their minimum distance. Therefore

$$a\sin i=1-x\ .$$

These equations, combined with those derived from the duration of the eclipse, give

$$a=3.191\ ;\quad i=3^\circ 14'\ .$$

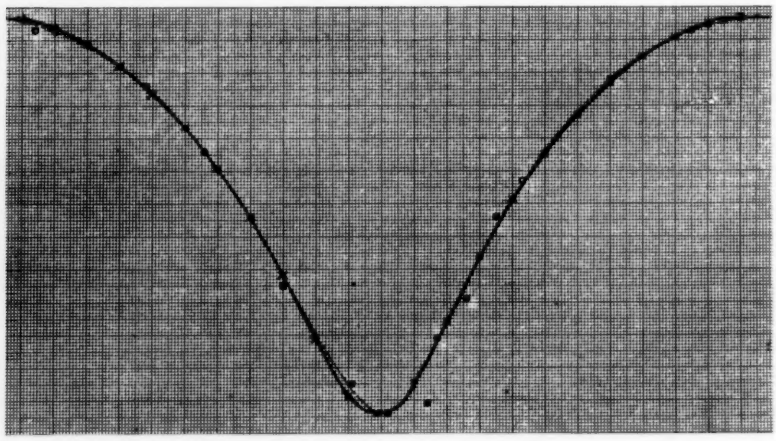
Since the light-curve C_o, given by P. and J., is not symmetrical the quantities for equal times before and after the minimum were combined into the mean values C_m. In the following table this mean curve is compared with the curves C_I and C_{II} calculated with the hypotheses I and II.

t	C_m	C_I	C_{II}	C_m-C_I	C_m-C_{II}	C_{III}	C_m-C_{III}
±0d00	7m64	7m640	7m640	0m000	0m000	7m640	0m000
.01	7.555	7.569	7.580	−.014	−.025	7.567	−.017
.02	7.38	7.428	7.403	−.048	−.023	7.385	−.005
.03	7.195	7.259	7.213	−.064	−.018	7.189	+.006
.04	7.02	7.090	7.044	−.070	−.024	7.014	+.006
.05	6.87	6.940	6.899	−.070	−.029	6.871	−.001
.06	6.745	6.810	6.778	−.065	−.033	6.749	−.004
.07	6.64	6.700	6.676	−.060	−.036	6.649	−.009
.08	6.565	6.608	6.592	−.043	−.027	6.566	−.001
.09	6.505	6.533	6.524	−.028	−.019	6.501	+.004
.10	6.47	6.476	6.472	−.006	−.002	6.455	+.015
±0.11	6.44	6.438	6.438	+.002	+.002	6.430	+.010

The change from hypothesis I to II decreases the residuals, but they retain the same sign. The values of x between 1 and 0.82 are therefore excluded.

The differences $C_m - C_{II}$ nearly disappear if a slight decrease is assumed in the duration of the eclipse. Since Professor Müller at Potsdam, in his provisional determination of the light-curve,[1] finds a considerably shorter duration, this change is probably justified. A few trials proved that a duration of $0^d.22 = 5^h\ 17^m$ (a decrease of only a quarter-hour) gave the best curve, C_{III} in the above table. As before, x was assumed as equal to 0.82, and therefore $a = 3.317$ and $i = 3°\ 7'$.

As the extremely small differences $C_m - C_{III}$ show, the theory represents the *mean* light-curve with great accuracy. But, as before



remarked, the original curve C_o is not symmetrical with respect to the minimum. This asymmetry, however, is in large part only apparent. If the minimum of the curve be assumed $0^d.0016$ earlier, the asymmetry almost disappears. Only in the immediate vicinity of the minimum can it still be seen, and there it is probably not real.

In the following table the curve C_o is compared with the theoretical curve, after the latter is shifted $-0^d.0016$.

In the accompanying illustration the theoretical curve C_{IV} is shown by the full line; the observed curve C_o, where it does not coincide with the former, is shown by the broken line. The small circles

[1] *Astronomische Nachrichten*, **171**, 359, 1906.

t	C_O	C_{IV}	C_O-C_{IV}	t	C_O	C_{IV}	C_O-C_{IV}
$-0^d.11$	$6^m.44$	$6^m.433$	$+0^m.007$	$+0^d.11$	$6^m.44$	$6^m.430$	$+0^m.010$
.10	6.48	6.461	+ .019	.10	6.46	6.449	+ .011
.09	6.52	6.510	+ .010	.09	6.49	6.492	− .002
.08	6.58	6.578	+ .002	.08	6.55	6.554	− .004
.07	6.66	6.664	− .004	.07	6.62	6.634	− .014
.06	6.77	6.767	+ .003	.06	6.72	6.731	− .011
.05	6.90	6.892	+ .008	.05	6.84	6.850	− .010
.04	7.05	7.040	+ .010	.04	6.99	6.988	+ .002
.03	7.22	7.219	+ .001	.03	7.17	7.159	+ .011
.02	7.40	7.414	− .014	.02	7.36	7.359	+ .001
.01	7.55	7.587	− .037	.01	7.56	7.549	+ .011
∓ 0.00	7.64	7.640	.000	± 0.00	7.64	7.640	.000

are the normal points which P. and J. find from the photographic plates. The residuals are as follows:

t	O.−C.	t	O.−C.	t	O.−C.	t	O.−C.
$-0^d.106$	$+0^m.03$	$-0^d.030$	$+0^m.03$	$+0^d.017$	$0^m.00$	$+0^d.043$	$-0^m.01$
− .072	− .01	− .009	− .05	+ .026	+ .04	+ .070	− .01
− .054	.00	+ .002	+ .01	+ .035	− .03	+ .095	.00
		[+ .014	+ .13]				

With the single exception of the normal point inclosed in the parenthesis,[1] the observations are satisfactorily represented by the theory.

We therefore find as the most probable elements:

Radius of the principal star = 1

$$x = 0.820$$
$$a = 3.317$$
$$i = 3^\circ\ 7'$$
$$(2t_0 = 0^d.22)$$

Even a considerable excentricity in the orbit has so slight an effect on the light-curve of an *Algol*-type star, that the relative size

[1] NOTE by translator.—The normal point at phase +0.014 depends upon only three images, as follows:

Plate	Exp.	Phase	Mag.	C_{IV}	O.−C.
R 1672...	5	0.012	7.61	7.49	$+0^m.12$
R 1650...	2	0.014	7.71	7.48	+0.13
UV 210...	7	0.017	7.52	7.47	+0.05

The large difference O.−C. seems mainly due to failure completely to reduce the observations with the reflector to the system of the more reliable extra-focal images.—J. A. PARKHURST.

and positions of the axes cannot be well determined from photometric data. It is hoped that the definitive reduction of the radial-velocity plates taken at the Yerkes Observatory[1] and Potsdam[2] will give more accurate values of these quantities.

According to Hartmann's provisional reductions, the distance a of the bright body from the center of gravity is 1,170,000 ($:\cos i$) km. If we assume that the orbit is circular and that the two components have the same mean density d, we derive the following elements:

Distance of centers = 3,297,000 km = 0.022 astronomical units.
Combined mass of the system = 1.002 sun's mass.
Mean density d = 0.222 sun's density.
Radius of the bright body = 994,000 km = 1.43 sun's radius.
Mass of the bright body = 0.646 sun's mass.
Radius of the satellite = 815,000 km = 1.17 sun's radius.
Mass of the satellite = 0.356 sun's mass.

SPECOLA VATICANA
March 1909

[1] *Astrophysical Journal*, **25**, 59, 1907.
[2] *Astronomische Nachrichten*, **173**, 101, 1906.

THE PHOTO-HELIOMETER

By CHARLES LANE POOR

In 1905 the writer published the first of a series of articles dealing with the "Figure of the Sun." In this paper were given the results derived from the measurement of a series of solar photographs taken by the late Lewis M. Rutherfurd in his private observatory in the years 1870–72. These early photographs were taken with a 13-inch lens photographically corrected and were made on collodion plates. These excellent photographs of Rutherfurd's, which have hardly been excelled at the present day, gave images of the sun with sharp, clean-cut, and easily measurable limbs. A determination of the polar and equatorial diameters gave results which were very accordant and which in precision compared very favorably with the heliometer measures made by the German observers of the transits of *Venus*. Several short series of more modern plates were measured, but on the whole their results were not as satisfactory as those obtained from the early plates of Rutherfurd. There is not a dearth of solar photographs in the world, for several observatories have for many years photographed the sun on each and every clear day, yet these photographs ordinarily are not suitable for accurate measurement. They were mostly made with horizontal telescopes, and were taken to supply data for investigations as to the number, size, and location of sun-spots. The mirror of such horizontal instruments introduces errors and makes the image unsymmetrical, and in the taking and developing of the photographs but little attention was paid to sharpness of edge.

In order to determine the best methods of applying photography to the determination of the size and shape of the sun, a series of experiments was undertaken and was carried out at the Yerkes Observatory by Mr. Philip Fox, under the direction of Professor Frost. A 4-inch photographically corrected lens of 40 feet focal length was mounted on the tube of the 40-inch Yerkes telescope. With this many photographs were taken, but none of them equalled in clearness and

sharpness of edge the early photographs of Rutherfurd. To a large extent this was due to the use of modern rapid dry plates, but some of the distortions of the image were undoubtedly caused by layers and currents of heated air near the tube of the great telescope.

In the summer of 1907 the 4-inch was dismounted and a pair of 2-inch photographically corrected lenses of 25 feet focal length were mounted in its place. These lenses were placed side by side in the same cell, so as to give overlapping images of the sun, and the cell was arranged so as to revolve about the collimation axis in a manner similar to the cell of a heliometer. The line joining the centers of the two lenses could readily be placed parallel to the equatorial or to the polar diameter of the sun, and the whole apparatus formed what might be termed a 25-foot photographic heliometer. Some trial photographs were made with the twin lenses by S. A. Mitchell of Columbia University (who was spending the summer at Yerkes) and by Mr. Fox.

There are two methods of utilizing the photographs made with such a heliometer, the first involving the measurement, on the overlapping images, of the width of the lune, the second requiring the measurement of the common chord.

1. The first method directly determines the length of the diameter, which passes through the centers of the two images. For, neglecting refraction, this diameter is equal to the known distance between the centers of the lenses added to the measured width of the lune. This is the simplest and most direct method of using a photographic heliometer; it obviates the use of a micrometer screw on the instrument and replaces an eye-estimate of contact by the measurement of a small distance on a photographic plate. The correction for refraction is extremely simple and may be made by the ordinary differential formulae.

2. In the second method the length of the diameter is only indirectly determined, and the corrections for refraction are somewhat troublesome and complicated. On each photograph the length of the chord, common to the two images, is a measure of the radius passing through the extremity of the chord, and slight changes in the radius produce large changes in the chord. As will be shown later, a change, Δr,

in the radius will produce a change, Δd, in the semi-chord approximately equal to

$$\Delta d = \frac{r}{d}\Delta r ,$$

where r and d are the respective lengths of the radius and the chord. Thus, if the chord be one-third the radius, an error of measurement of 1″ in the chord would produce an error of only 0″.3 in computed length of the radius. This is the great advantage of the second method.

The ultimate choice of the two methods depends upon the relative accuracy with which the different measures can be made. In order to test this point, the chord and the lune were measured on twelve test plates. The measures of one of these were rejected as on that plate the width of the lune could be only approximately determined, because "the background was too dark to distinguish the edges of the lune." The probable errors of a single measure were found to be:

	Lune	Chord
Harpham.....	±0.011 mm	±0.011 mm
Davis.....	±0.015 mm	±0.011 mm

This would seem to show that the chord can be measured with as great accuracy as the lune and would therefore indicate a decided superiority in this method. This chord not only furnishes an extremely accurate and delicate measure of the radius, but it is also extremely easy of measurement. The measures can be made far more rapidly than those necessary on a direct photograph of the solar disk. Unfortunately, however, as has been noted, the reductions for refraction are rather complicated, and special formulae are necessary.

Aside from the general effect of bad seeing outside the 40-inch tube, the photo-heliographic method suffers special defects from bad seeing. When watching the solar image, pulsations of bad definition can be seen to sweep across the solar disk. Entering at one limb, they can be followed across the disk by the rippling along the limb, so that at any moment one part of the limb may be steady, and other parts unsteady. In the overlapping images of the photo-

heliograph, two points of one limb fall upon two points of the opposite limb, and to have a perfect photograph these four points of the limb must have sharp definition. It is not difficult to get one image in the common area sharp, but to get both sharp requires unusual steadiness. Any distortion of one image in the points of intersection will increase or decrease the length of the common chord, and will give an erroneous value for the radius. The distortion will be a change in length of the solar radius, and this will increase or decrease the length of the chord. As the measured length of the chord is a delicate test of the length of radius, according to the formula:

$$\Delta d = \frac{r}{d}\Delta r,$$

any error in its length will multiply the error in determining the radius, and in just the ratio $\frac{r}{d}$.

In measuring a simple solar image, such local errors from bad seeing are not magnified, and may moreover be eliminated by taking the mean of several measures of the diameter near the equatorial and axial diameters. Here we have not this possibility, and to offset the disadvantage, more plates must be measured than in the older method. On account of the simplicity of measurement and reduction, however, this is not a costly expedient.

REFRACTION FORMULAE

Let $L'L''$ be the two lenses mounted in a single cell at a fixed distance apart and with the optical axes $L'C'$ and $L''C''$ parallel. The cell with the two lenses can be revolved about the common collimation axis LC.

Each lens will form an independent image of the sun, and these images will appear on the photographic plate, P, with their respective centers at C' and C''. If the adjustment of the lenses is perfect, each image will be an exact duplicate of the other, and if there be no refraction they will be sensibly circular. The distance between the centers of the two images, which will be equal to that between the optical centers of the two lenses, is one of the constants of the

instrument, and will be denoted by $2a$. From the measured value of the common chord, $2d$, the radius of the image, passing

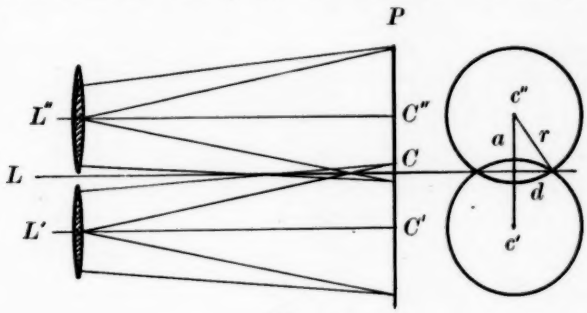


FIG. 1

through the extremity of the chord, can be readily found by the formula,

$$R=\sqrt{a^2+d^2}\,. \qquad (1)$$

Refraction deforms the apparent disk of the sun into a figure of unknown shape which, however, is symmetrical with respect to the

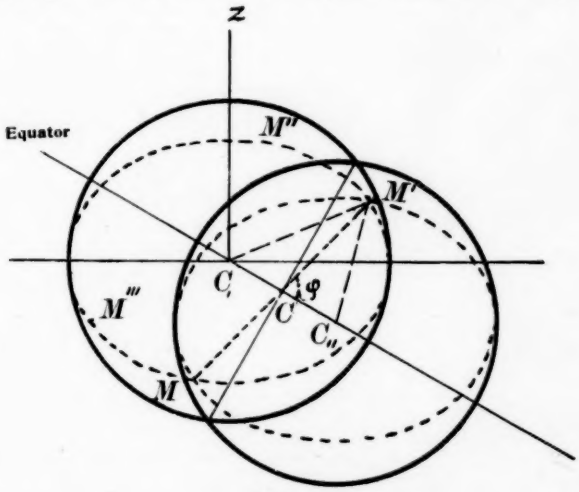


FIG. 2

vertical axis through the center. The real images which appear on the photographic plate are therefore two symmetrical but unknown curves (approximately ellipses) whose centers are at $C_,$ and $C_{,,}$. These deformed images are shown in Fig. 2, which represents

the case when the line joining the centers of the two lenses is parallel to the equator of sun. The image formed by lens I has its center at $C_{,}$, the image formed by lens II is identical with that formed by I, but it has been moved, as it were, along an axis parallel to the sun's equator until its center is at $C_{,,}$. The common chord of the two images is the line MM', which passes through C, the point in which the collimation axis of the instruments cuts the focal plane. The ends of the chord, M and M', do not correspond to real points in the heavens, they are merely the intersections of the two curves in the focal plane, or on the photographic plate. The points of the two curves, which coincide on the image, and form the the point M, in reality lie on opposite sides of the sun. It is thus incorrect to speak of "the refraction" of the point M; yet this point of intersection is changed by the effects of refraction, changed by the combined effects of the refraction of the points M and M'''.

If the sun be photographed at a very small altitude, then the refractions of the four points M, M', M'', and M''' will be different. The radius CM, for example, is not only shortened by refraction, but it is also turned through a minute angle. The entire refraction effect is differential, and formulae for finding the actual amount of the differential refraction both in radius and position angle were developed by Bessel. These formulae require the use of the position angle at the middle point of the distance measured and, therefore, are not entirely suitable for photographic plates. Jacoby made a slight modification in these formulae and adapted them for photographic work. They are:

$$\begin{aligned}\sigma - s &= sx\,[\tan^2 \xi \cos^2 (p-q) + 1] + \text{terms of second order}\\ \pi - p &= -x \operatorname{cosec} 1'' \tan^2 \xi \sin (p-q) \cos (p-q)\\ &\quad -x \operatorname{cosec} 1'' \tan \xi \sin q \tan \delta\\ &\quad + \text{terms of second order,}\end{aligned}$$

in which s and p are the measured distance and position angle, and σ and π the same corrected for refraction. These formulae of Jacoby are identical with those of Bessel, with the exception of the terms of the second order, which terms are usually so small as to be entirely inappreciable. The term:

$$-x \operatorname{cosec} 1'' \tan \xi \sin q \tan \delta\,,$$

which affects all the position angles alike on any given plate, may be neglected and will therefore be omitted from the above formula.

At opposite ends of a diameter the second-order terms are equal, but of contrary sign. They only begin to become appreciable, however, when the altitude of the sun is less than 26°, at which point these terms may amount to as much as $\pm 0''.013$. At altitudes greater than this, therefore, the refractions of M', and M''' are sensibly equal, as are also those of M'' and M. Hence at altitudes greater than 26°, refraction deforms the solar image (assumed to be a circle) into a curve which is sensibly symmetrical with respect to the horizontal as well as the vertical axis.

At about 17° altitude the second-order terms become as great as $\pm 0''.05$, and at smaller altitudes the terms increase very rapidly. The figure is no longer symmetrical with respect to the horizontal axis and the common chord MM' no longer bisects the line $C'C''$. But for all practical altitudes it may be assumed that these two lines mutually intersect; the error in this assumption being only of the third order. Denoting the refraction of the radii drawn to M, M', M'', and M''' by r, r', r'' and r''' respectively, we shall have for the true lengths of these radii,

$$\begin{aligned} C,M &+r \\ C,M' &+r' \\ C,M'' &+r'' \\ C,M''' &+r'''. \end{aligned}$$

And without sensible error we may take,

$$\begin{aligned} r &= r'' \\ r' &= r''' . \end{aligned}$$

Further, if the sun itself be regarded as a figure of revolution about its polar axis, then radii symmetrically placed with respect to the equator will be equal, and the four above-mentioned radii will all be equal. We have, therefore,

$$C,M'+r'=C,M''+r''=R ;$$

but $C_{,,}M'$ is C,M'' moved into a new position by the action of the second lens. Hence we have

$$C,M'+r'=C_{,,}M'+r''=R , \qquad (2)$$

where R is the true length of the sun's radius.

From this we find at once

$$C_{,}M'-C_{,,}M'=r''-r'. \tag{3}$$

Now in the two triangles $C_{,}M'C$ and $C_{,,}M'C$ the sides $C_{,}C$ and $CC_{,,}$ are equal, each being one-half the distance between the optical centers of the two lenses. This is a constant of the instrument and has already been denoted by a. The distance CM' is one-half the measured chord and is denoted by d. The angle at which $C_{,}C_{,,}$ and $M'M$ intersect will be denoted by ϕ; it differs but slightly from a right angle.

From trigonometry we have:

$$\overline{C_{,}M'}^{2}=a^2+d^2+2ad\cos\phi$$

$$\overline{C_{,,}M'}^{2}=a^2+d^2-2ad\cos\phi\,,$$

whence by addition,

$$\overline{C_{,}M'}^{2}+\overline{C_{,,}M'}^{2}=2(a^2+d^2)\,. \tag{4}$$

By solving equations (3) and (4) the values of $C_{,}M'$ and $C_{,,}M'$ can be found as follows:

$$C_{,}M'=\tfrac{1}{2}(r''-r')+\sqrt{a^2+d^2-\tfrac{1}{4}(r''-r')^2}$$

$$C_{,,}M'=-\tfrac{1}{2}(r''-r')+\sqrt{a^2+d^2-\tfrac{1}{4}(r''-r')^2}\,. \tag{5}$$

Whence from (2) the length of the radius will be:

$$R=\tfrac{1}{2}(r''+r')+\sqrt{a^2+d^2-\tfrac{1}{4}(r''-r')^2}\,, \tag{6}$$

and this determines the corrected radius in terms of the refractions and the measured chord.

The radius thus found, however, is not the equatorial radius, nor is it any single radius. It is the mean value of two radii, symmetrically placed on each side of the equator. The angle at which these radii are inclined to the equator may be found with sufficient accuracy by taking

$$\tan\theta=\frac{d}{a}\,. \tag{7}$$

This assumes the chord to be perpendicular to the line joining the centers, and therefore neglects the slight tilting effect of refraction. This tilting effect amounts to only a few minutes of arc and the consequent error in θ is small.

If the centers between the two lenses should be separated to such an extent that the two images become tangent, then the chord vanishes

and the instrument becomes a heliometer of the ordinary form. In this case the refractions r' and r'' are equal and formula (6) reduces to,

$$R=a+r'. \tag{8}$$

In this case a is variable and is measured by a micrometer screw. An error in the measurement of a will directly effect R to its full value.

In the photographic method a is constant and d is measured. The effect of an error in measuring d upon the resulting value of R can be found by differentiating (6), regarding a and the refractions as constant as follows:

$$\Delta R=\frac{2d\cdot\Delta d}{2\sqrt{a^2+d^2-\frac{1}{4}(r''-r')^2}}.$$

But, neglecting refraction, the value of the denominator of the above fraction is equal to R: hence, as heretofore given, we have:

$$\Delta R=\frac{d}{R}\Delta d \text{ (approximately).} \tag{9}$$

Therefore an error in measuring d will affect R only in the ratio $\frac{d}{R}$. If the half-chord subtend an angle of $5\frac{1}{2}°$ then $\frac{d}{R}=0.1$, and an error of measurement will affect the result by only $\frac{1}{10}$ of its amount. In the test photographs made at the Yerkes Observatory the scale was such that one millimeter was approximately equal to $27''.0$ and the chord was one-third the radius. Hence an error of measurement of 0.01 mm in the chord would cause an error of only 0.0033 mm, or $0''.10$ in the computed value of the radius. On good photographs quantities of this size can be easily measured.

EXPERIMENTAL RESULTS

As an example of the reductions I will refer to the plates taken at the Yerkes Observatory by Mr. Fox on November 12, 1907. The data, as furnished by him, follow:

Plate	Σe 137	Σe 138
Date	November 12, $10^h\ 58^m.7$	$11^h\ 2^m.3$
Diameter	Equatorial	Polar
Barometer	29.33 in.	29.33
Att. Thermometer	48°.5	48°.5
External Thermometer	31.0	31.0

The plates were measured with the Repsold measuring machine of

Columbia University, on different days and by different observers, with the following mean results:

	Σe 137	Σe 138
Harpham	32.686	32.560
Poor	.670	.540
Davis	.698	.547
Mean	32.6847	32.5490
d	16.3424	16.2745

From measures on a series of plates a preliminary value of the distance between the centers of the two lenses was obtained. This gave for the half distance, or instrumental constant,

$$a = 31.9424$$

and from the formula,

$$R = \sqrt{a^2 + d^2},$$

an approximate value of the apparent radius may be calculated for each plate. Then from the given data can be computed the zenith distance of the center of the sun at the time each plate was taken, and thence, with the approximate values of the apparent radius, the refractions for the points of intersections. We thus find:

	Σe 137	Σe 138
Zenith distance	60° 47′ 10″	60° 40′ 3″
Parallactic angle	− 8° 21′ 19″	− 7° 36′ 28″
θ	27° 5′ 17″	27° 3′ 45″
Approximate radius	35.8803 mm	35.8493 mm
Refraction, r''	+ 0.0330 mm	+ 0.0422 mm
" r'	+ 0.0102 mm	+ 0.0200 mm
$(r''-r')$	+ 0.0228 mm	+ 0.0222 mm
$\frac{1}{2}(r''-r')^2$	+ 0.0001 mm	+ 0.0001 mm
$\frac{1}{2}(r''+r')$	+ 0.0216 mm	+ 0.0311 mm

From formula (6) now are found the corrected values of the radii, and reducing the results to distance unity, we have finally:

Equatorial radius, plate Σe 137 = 35.5296 mm
Polar radius, plate Σe 138 = 35.5082 mm
Mean radius = 35.5189 mm
Polar minus equatorial = −0.0214 mm

The scale value was approximately determined as:

$$1 \text{ mm} = 27''.06.$$

And this, combined with the above value for $P-E$, shows that on this

date the radius measured on the equatorial plate exceeded that on the polar plate by

$$0''.57 .$$

These radii cut the surface of the sun in latitudes 26° and 64° respectively.

A series of similar plates taken on various days during October and November give the following results:

Date	Mean Radius	Residuals		*P*−*E* Arc
		mm	Arc	
Oct. 18.....	35.538	+033	+0".89	−0".81
21.....	.469	−036	−0.97	−1.16
22.....	.497	−008	−0.22	−1.43
23.....	.536	+031	+0.84	−1.08
Nov. 8.....	.469	−036	−0.97	−0.62
12.....	.519	+014	+0.43	−0.57
Means.....	35.505			−0.95

CONCLUSIONS

The photographic tests made at the Yerkes Observatory establish the value of the photographic method and of the photo-heliometer, but they showed that in order to obtain the most satisfactory results several essentials must be observed:

1. The results derived from the use of the photo-heliometer are more satisfactory and accurate than those derived from direct photographs of the sun. The measurement of the chord common to the overlapping images can be made more easily and rapidly than can the measurements of the diameter on a direct image of the sun. And again, the reductions and computations connected with a heliometer measure are very much more simple than are those necessary to determine the relative lengths of the diameters from measures upon a direct photograph of the solar disk. It takes far longer to measure and reduce a number of diameters on a direct photograph of the sun, than it does to measure and reduce an equal number of heliometer images. The defects from bad seeing, heretofore mentioned, can be eliminated by taking the mean of a number of plates made on the same day. Hence for the purpose of determining the relative lengths of various diameters of the sun, and of possible variations therein, the

photo-heliometer would seem to be more efficient than the direct photographic telescope.

2. The photographic lenses, or half-lenses of the heliometer, must be carried on a separate and specially designed equatorial mounting, so constructed that the shutter is independent of the mounting. If attached to the mounting, the shutter will almost invariably cause jars and consequent distortions of the image. To obtain images of sufficient size for accurate measurement the focal length of the objective must be comparatively great; from twenty-five to fifty feet. To satisfactorily mount and house such a large instrument is a complicated and very expensive matter. The use of such a large and cumbersome instrument might be overcome by the use of a fixed magnifying system, somewhat similar to tele-photo lenses. By making the optical system fixed, so that the same magnification would always be obtained, many of the objections to the use of a magnifying eyepiece would be overcome, and the total length of the instrument kept within reasonable and handy limits. Simple photographic telescopes, using this principle of magnification and made by Steinheil, have been in use for some time.

3. Wet plates should be used. On the modern rapid dry plates the edges of the image are not as sharp and clear as those on the old collodion plates.

COLUMBIA UNIVERSITY
1908

Minor Contributions and Notes

NOTE ON ATMOSPHERIC TRANSMISSION BY NIGHT AND BY DAY

M. Ch. Nordmann, of the Paris Observatory, announces (*Transactions of the International Union for Co-operation in Solar Research*, **2**, 48, 1908) that "in many cases, especially when it is very clear, the distribution of atmospheric absorption in the spectra of the stars appears to be notably different from what it is when studied in the solar spectrum." He says: "I am able to state in the majority of cases and when the sky is clear, that for zenith-distances less than about 70° (because beyond this point the phenomenon returns to the diurnal type), the violet and blue parts of the spectrum are notably less absorbed at night than by day. I have found that, in certain cases, this part is less absorbed than the red part itself." He concludes that the atmospheric absorption is different at night from what it is in the daytime.

In regard to M. Nordmann's hypothesis of a difference in the absorption of short waves by day and by night, the following considerations show that it cannot be admitted: The human eye has an extraordinary range of sensitiveness for different colors. For example, my own eye is 790,000 times as sensitive for green light of wave-length $0.5\,\mu$, as it is for deep-red light, $\lambda = 0.768\,\mu$. Moreover, although the change is not as rapid in the brighter part of the spectrum, there is still a variation of sensitiveness in the ratio 1:10 in passing from $\lambda = 0.60\,\mu$ to $\lambda = 0.55\,\mu$. Consequently, differences in the distribution of the more intense groups of absorption lines, such as occur in the spectra of stars of different types, may shift the point of maximum brilliancy into regions of the spectrum for which the eye has very diverse degrees of sensitiveness. A comparatively slight change in the wave-length of the ordinate of maximum brightness in the spectral luminosity-curve, due to absorption of the rays by the atmospheres of either the star or the earth, may thus correspond to a much more considerable change in the star's apparent brightness on account

of the unequal sensitiveness of the eye to the combined rays when those wave-lengths to which the eye is especially sensitive are either deficient, or present in excess. Thus the change of apparent brightness for the same difference of altitude may vary considerably in the case of two stars whose original light-distribution is not the same in the region of the spectrum which chiefly contributes to the visual impression. Much larger changes in other parts of the spectrum, such as the violet, which experience strong absorption, but do not affect the eye much, will have relatively little effect on the apparent magnitude of the star obtained by eye estimate, using the sum of all visible rays. Thus the apparent transmission of star light, determined photometrically, is much larger than the combined transmission of solar luminous rays deduced from spectrobolometric observations at high and low altitudes, the reason being that, while the stellar observation ostensibly relates to luminous rays of every wave-length, it actually or virtually refers to a limited region of the visible spectrum which has a larger transmission-coefficient than the average for all kinds of light. Some incongruity of light-distribution in the region of the spectrum to which the eye is most sensitive is probably responsible for discrepancies which have been noted in photometric observations of stars of different types at high and low attitudes.

The visual phenomena described above, however, do not affect photometric comparisons made in the spectrum. Here another cause is at work; for by night and in feeble light, the eye is much more sensitive to violet than by day, and is relatively much more sensitive to light of enfeebled intensity in the violet than to equally enfeebled green or yellow light, whenever the luminous intensities are equally dim. This is due to a physiological cause which appears to be connected with the quite gradual formation of the violet pigment which assists in the absorption and thence in the recognition at the retina of short waves, and which is more or less destroyed by exposure of the eye to strong daylight. Thus a rest of 15 or 20 minutes in complete darkness is required to restore the sensitiveness of the eye to violet light, whereas a much shorter period suffices for the longer light-waves. Hence the apparent increase of atmospheric transmission of violet rays by night, noticed by M. Nordmann, finds its explanation in physiological causes. It will be noted that as the star sinks lower

and becomes fainter, the eye becomes relatively more sensitive to violet light; but even this is insufficient to reverse the great absorption of violet rays at very low altitudes of the stars, since M. Nordmann finds that when the zenith-distance exceeds 70°, the ordinary daylight law is restored. In photometric comparisons made at night in the lunar spectrum where there is abundance of light, I find no other change at high and low altitudes of the moon, and no other law of atmospheric absorption than that obtained from solar observations in the daytime, which includes minor variations, but never a reversal of the order of absorption for violet and red.

FRANK W. VERY

WESTWOOD, MASS.
March 31, 1909

THE ARC SPECTRUM OF IRON

The article on the arc spectrum of iron, λ 6855 to λ 7412, by E. J. Evans, published in the March number of this Journal, does not refer to any earlier measurements of the iron lines in the region of longer wave-lengths than λ 6855. I may therefore be permitted to say that a careful measurement of the portion of the iron spectrum in question, extending to λ 7600, was made in a thesis prepared under Professor Kayser's direction, which perhaps escaped the attention of Mr. Evans on account of the place of its publication. The reference is: E. A. Kochen, *Der rote Teil des Eigenbogenspectrums*, pp. 21. Dissertation, Bonn 1909; *Zeitschrift für wissenschaftliche Photographie*, **5**, 285–299, 1907.

H. KONEN

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NOTICE OF INCREASE IN SUBSCRIPTION PRICE

With the thirtieth volume of the *Astrophysical Journal*, beginning with the July number, 1909, the annual subscription price will be increased by one dollar, thus becoming five dollars, or $2.50 per volume of five numbers. This action becomes necessary as the result of the increasing cost of production, particularly in respect to printing and engraving. The editors have reluctantly yielded to the necessity of this change, although, in view of the number of pages per year and of the quantity and quality of the engravings, the *Journal* has been relatively much cheaper than its contemporaries foreign or domestic. An increase cannot be asked in the subsidy granted by the University of Chicago, amounting to about 60 per cent. of the cost of publication. The editors believe that the subscribers will prefer to have the subscription price raised rather than to have the size of the *Journal* reduced or the average number and quality of the illustrations diminished.

Particulars as to the price of foreign subscriptions and of single numbers will be found in the publishers' notice elsewhere in this number.

THE EDITORS

YERKES OBSERVATORY
April 1909